Homoclinic chaos and Mixed-mode oscillations

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Hands School on Nonlinear Dynamics 2015
IPR, Gandhinagar

Outline

1. Homoclinic Bifurcation

2. Shilnikov Chaos via Mixed-mode oscillation

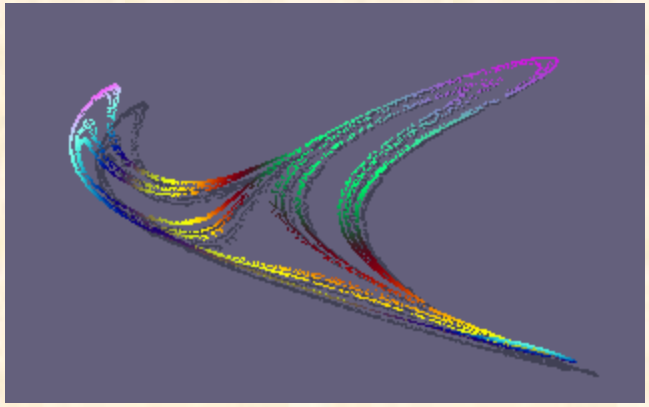
- Experiment: Chua circuit

3. Gluing bifurcation

- Experiment: Chua circuit

Dana, Chakraborty, Ananthakrishna, *PRAMANA-J.Phys.* **64** (3), 443 (2005)
Roy, Dana, *Int. J. Bifur .Chaos* **16** (12), 3497 (2006)
Dana, Roy, *Int. J. Bifur .Chaos*, **17** (10), 3437 (2007)
Chakraborty, Dana, *Chaos* **20**, 023107 (2010)

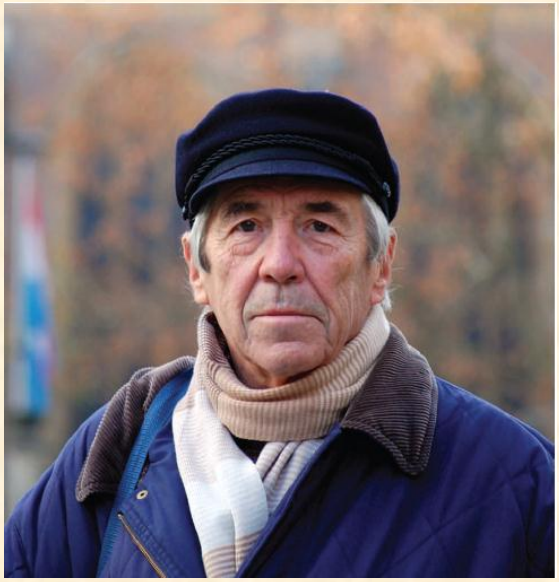
Stretching and Folding



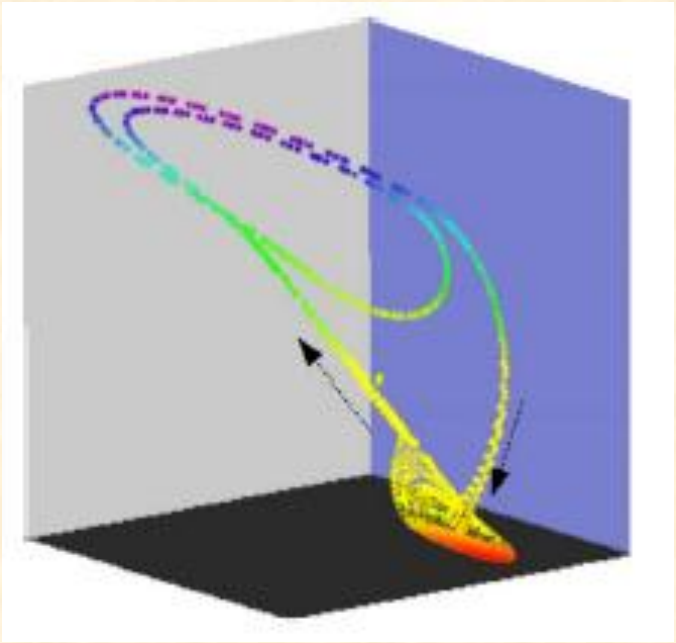
3D system: Lyapunov exponents

λ_1=+ve : Stretching
λ_2 =-ve : Folding
λ_3=0 : no expansion of phase

Shilnikov Chaos



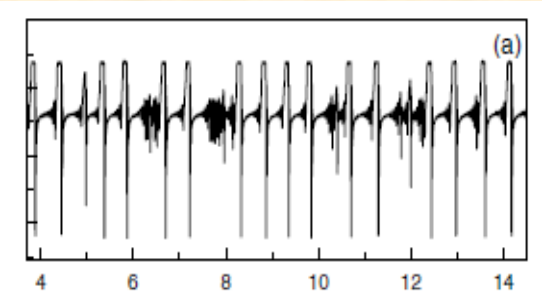
Leonid Pavlovich Shilnikov
(17th Dec., 1934 - 26th Dec., 2011)

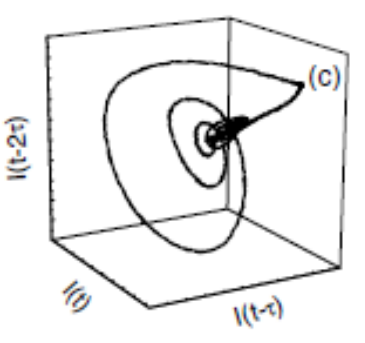


L.P.Shilnikov, *Sov.Math.Dokl*. 6:163-166 (1965)

Homoclinic chaos

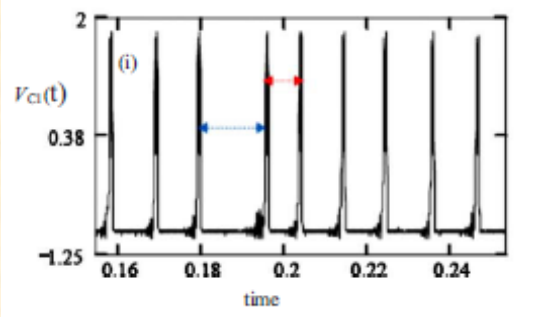
CO_2 Laser

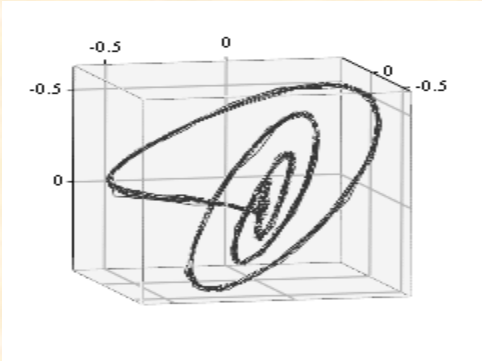




F.T.Arecchi *et al*.,
Phy.Rev.Lett. **86** (2001)

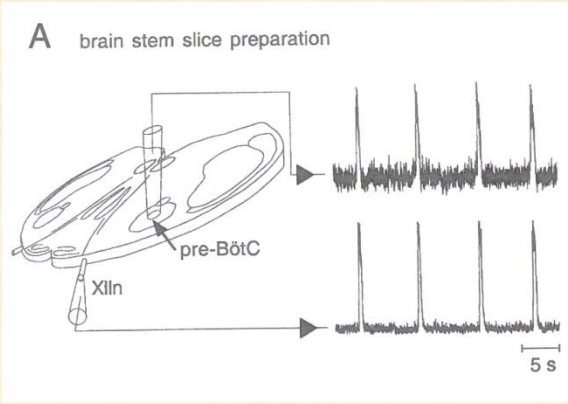
Chua Circuit





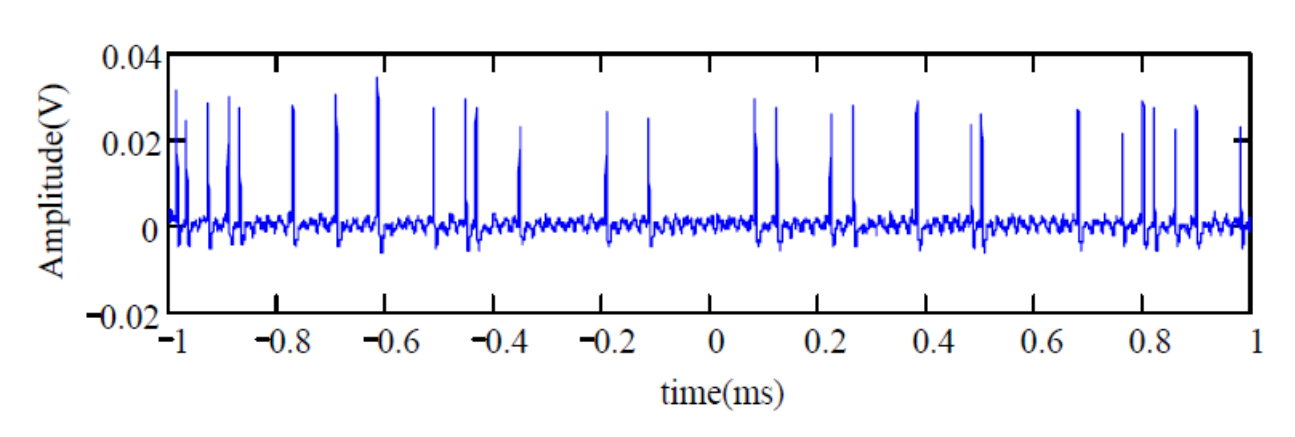
S.Chakraborty, S.K.Dana
Chaos, **20,** 023107 (2010)

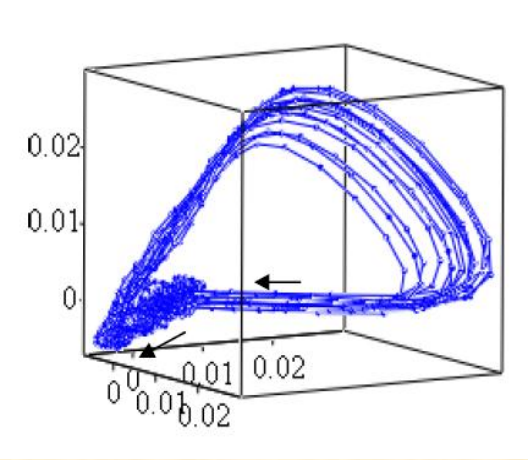
Neuronal Dynamics



C.A. Del Nigro et al**,** Biophy.J., **82,** 206-214, (2002)

Plasma Discharge

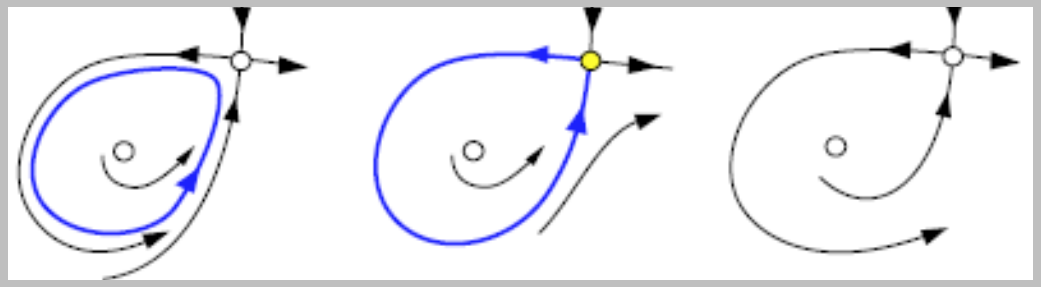




S. Chakraborty, A. N. S. Iyengar,
SINP, Kolkata, 2013

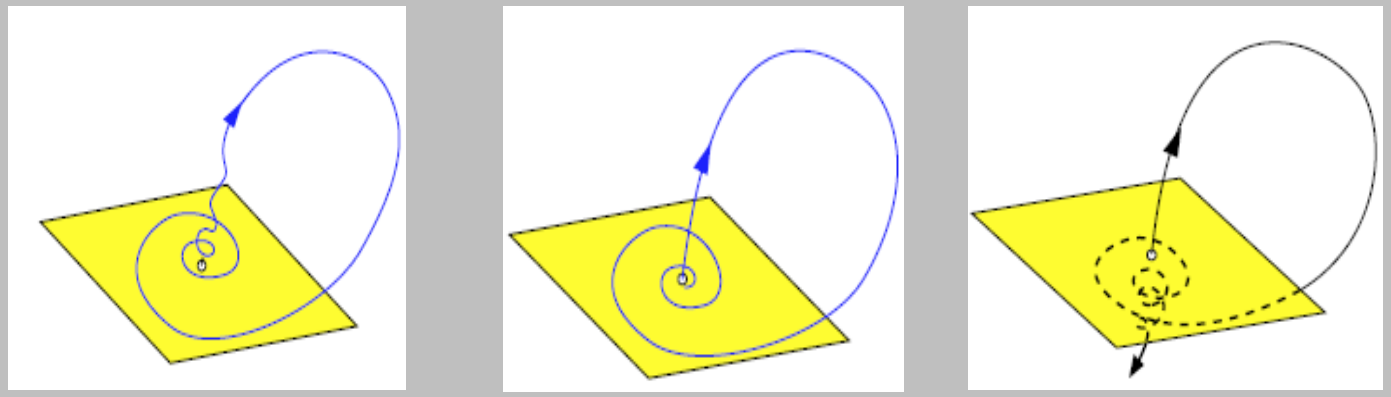
Homoclinic Bifurcation

(Global bifurcation)



2D system
Saddle

Saddle: $\gamma_1, -\gamma_2$



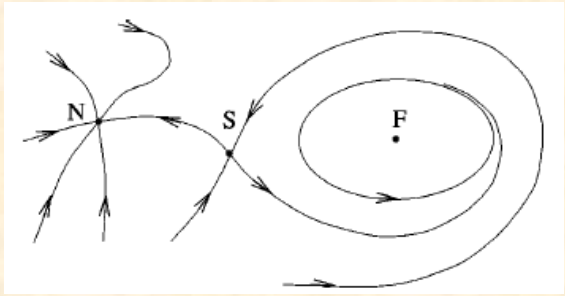
3D System
Saddle
Focus

Saddle focus: $\gamma, -\sigma \pm j\omega$

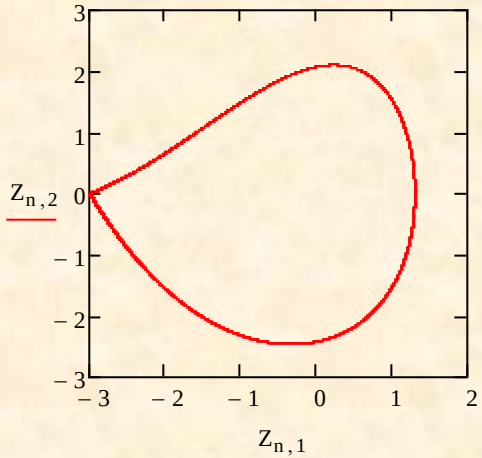
Modified van der Pol system (Mathcad Demo)

$$\dot{x} = y$$

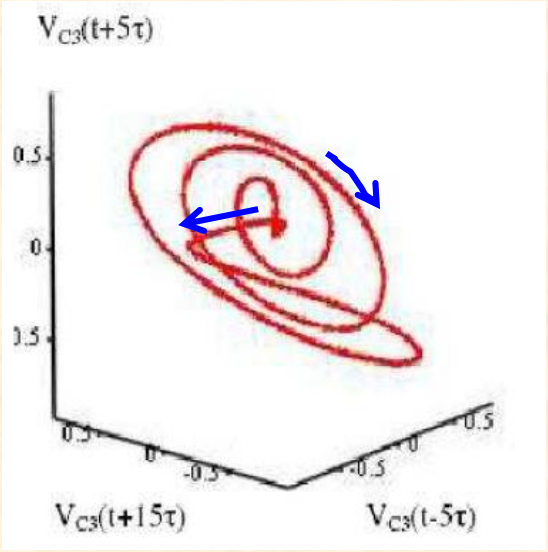
$$\dot{y} = \alpha(\mu - x^2)y - \frac{x(x+d)(x+2d)}{d^2}$$

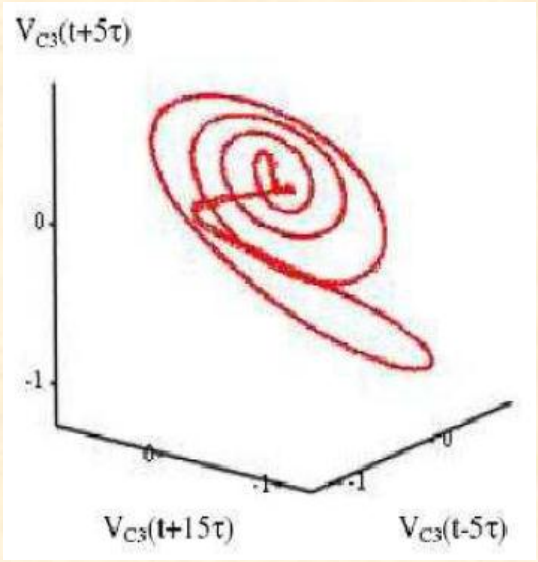


Node: $-\alpha_1, -\alpha_2$; **Saddle:** $\gamma_1, -\gamma_2$; **Saddle focus:** $\gamma, -\sigma \pm j\omega$



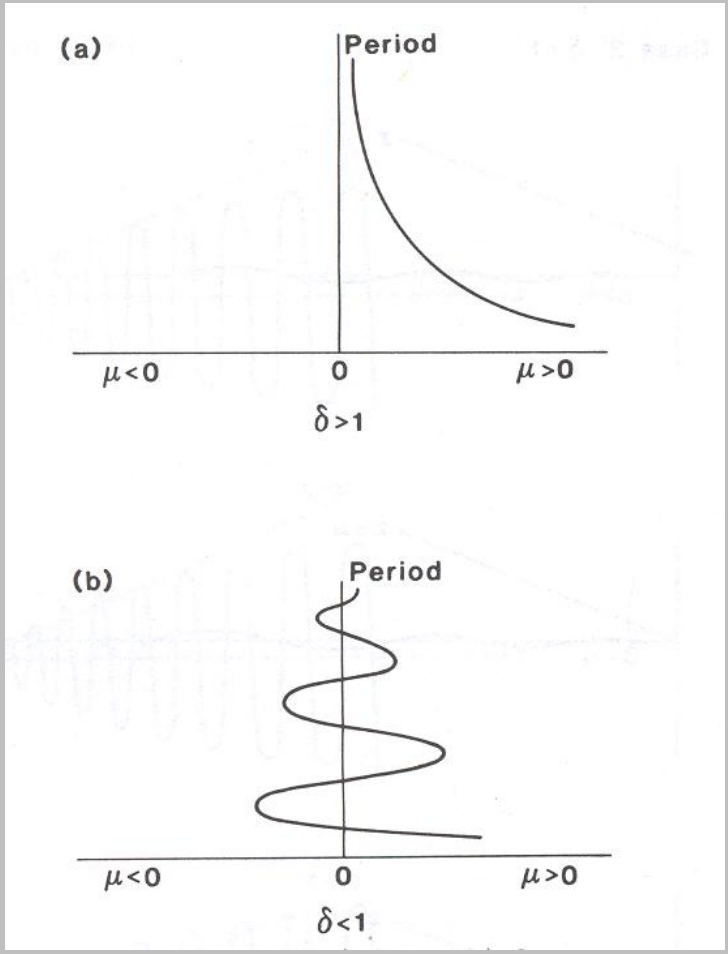
Limit cycle close to Homoclinic point

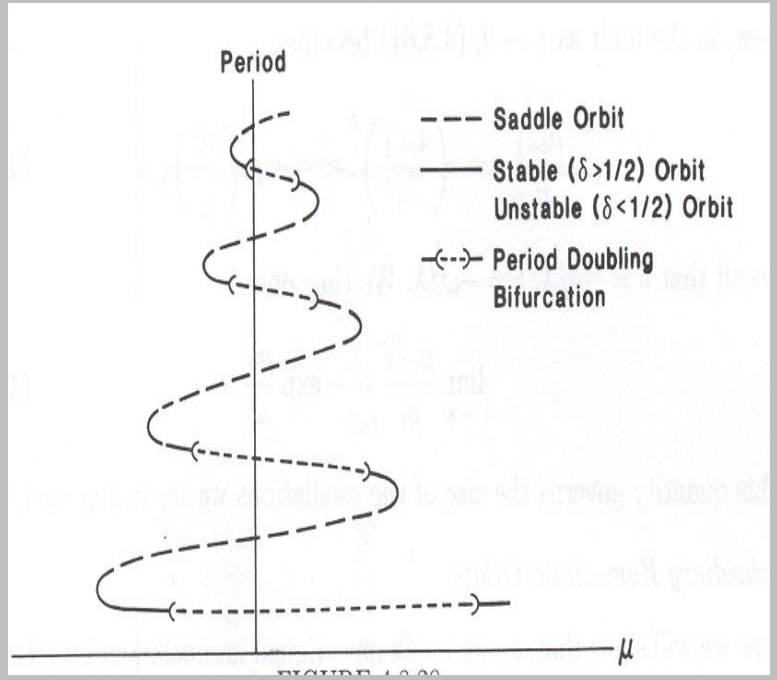




Saddle Focus: $\gamma, -\sigma \pm j\omega$

Period-Parameter Bifurcation

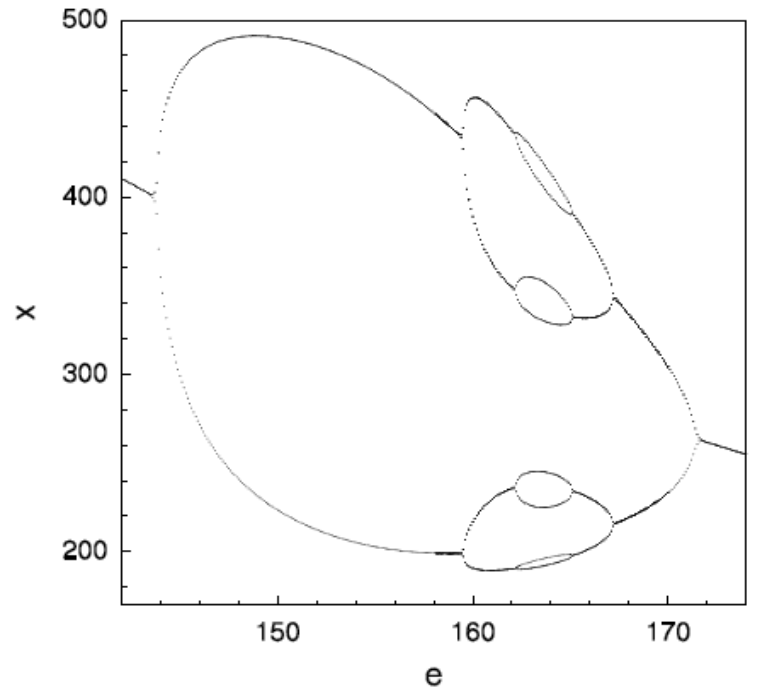


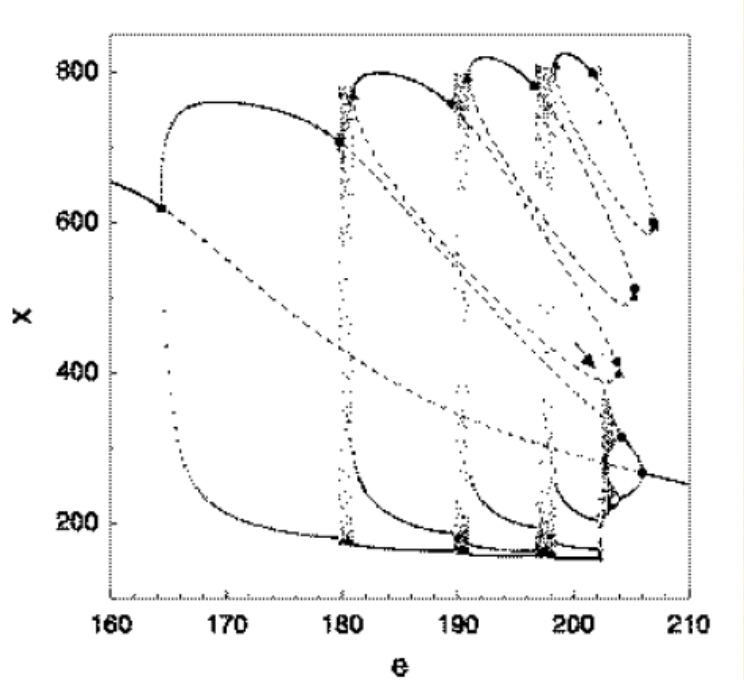


$\delta = |\gamma/\sigma|$

Regimes of Homoclinic Chaos

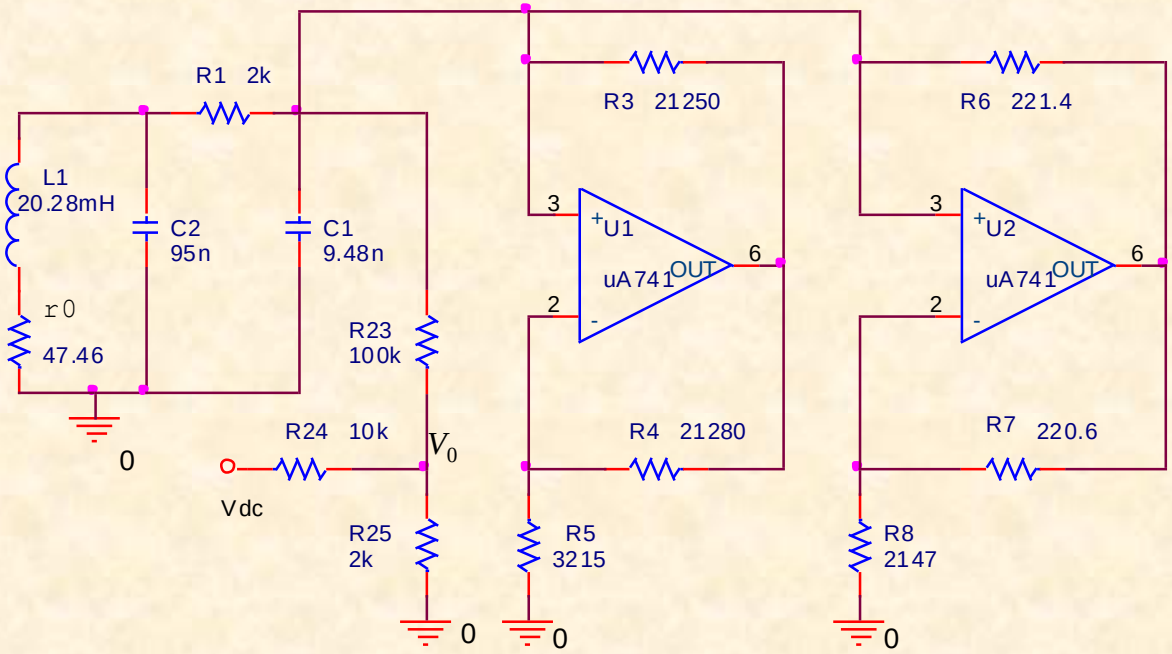
Slow-fast dynamics





Experiment: Chua circuit

Single Chua Circuit: Asymmetry-induced



$$V_0 = V_{dc}\frac{R_{23}}{R_{24}+R_{25}}$$

$$R_p = R_{23}$$

Inverse-symmetry

$$f(V_{C1}, V_{C2}, I_L) = f(-V_{C1}, -V_{C2}, -I_L)$$

$$\frac{dV_{C_1}}{dt} = \frac{1}{R_1 C_1}[(V_{C_2} - V_{C_1}) - R_1 f(V_{C_1})]$$

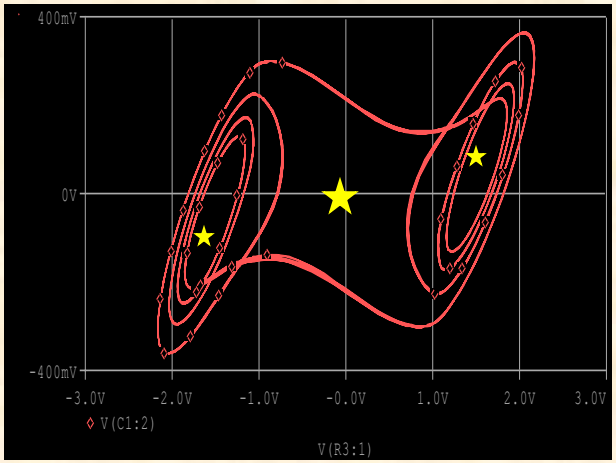
$$\frac{dV_{C_2}}{dt} = \frac{1}{R_1 C_2}(V_{C_1} - V_{C_2} + R_1 I_{L_1})$$

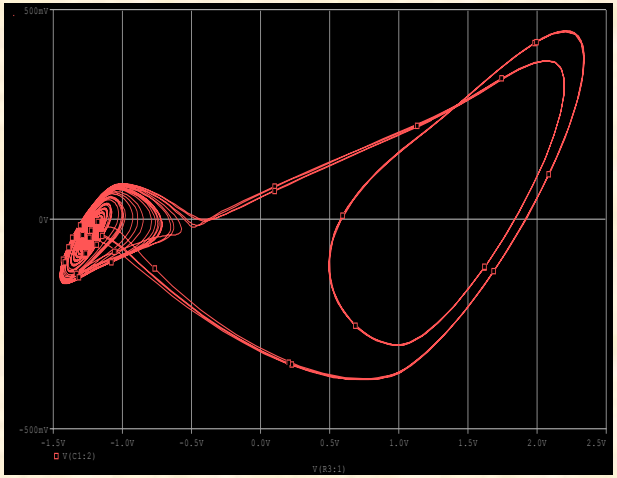
$$\frac{dI_{L_1}}{dt} = \frac{1}{L_1}(-V_{C_2} - r_{01} I_{L_1})$$

$$+\frac{1}{C_1 R_p}(V_0 - V_{C_1})$$

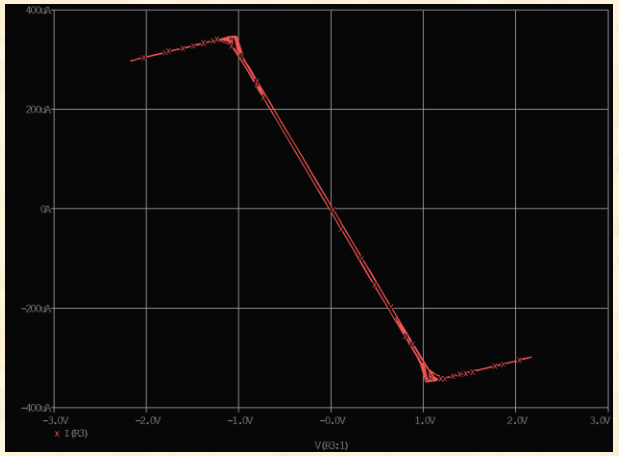
Double Scroll Attractor

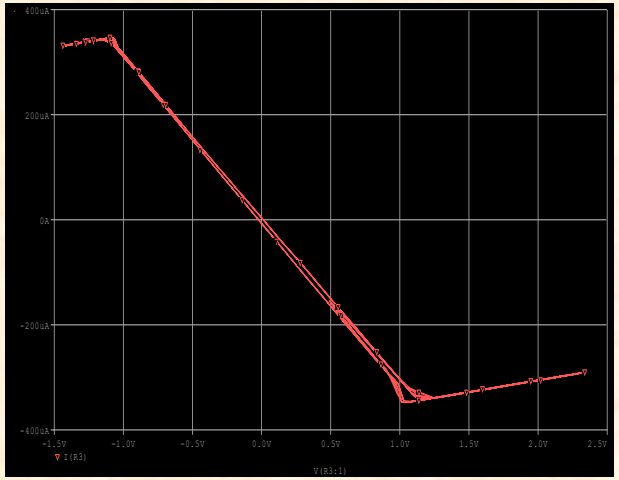
Phase Portrait





Piecewise Linear Function

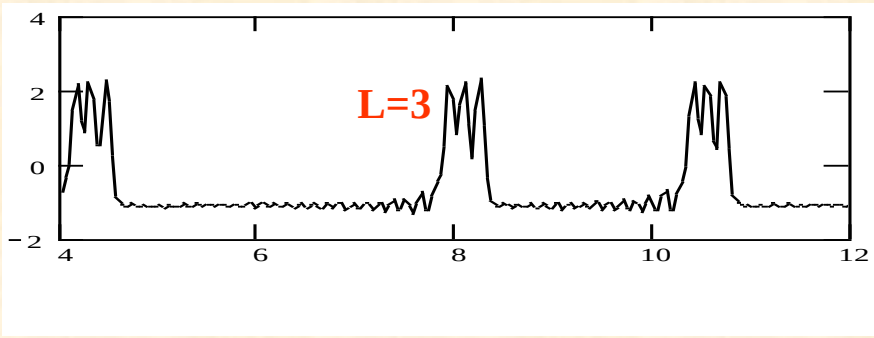




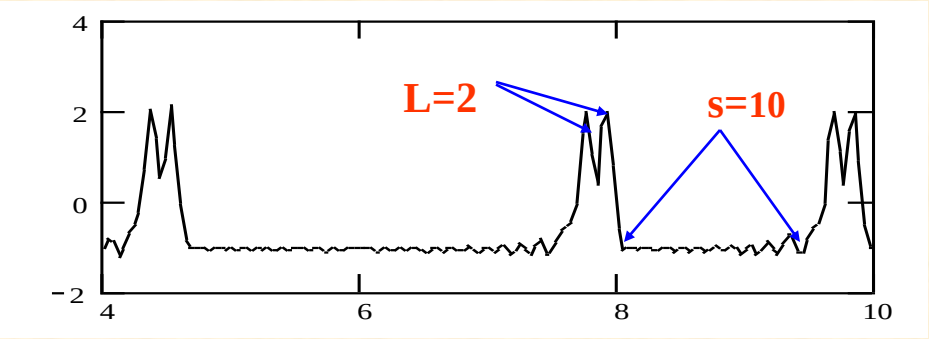
Shilnikov Chaos

L^{∞}

Supercritical Hopf

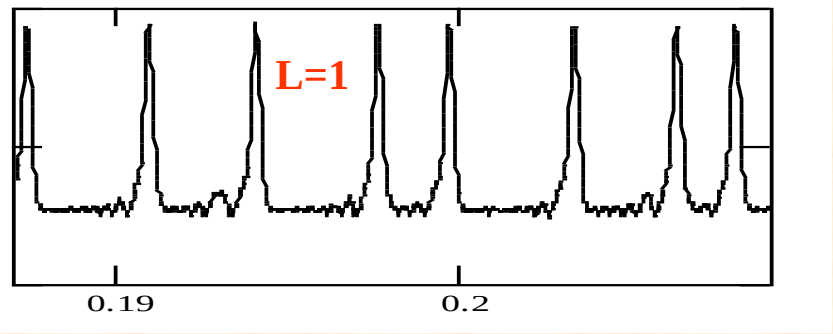


3^{∞} :R_1=1386Ω,
R_8=1833Ω,
R_C=95kΩ



2^{∞} :R_1=1427Ω,
R_8=1954Ω,
R_C=101kΩ

R_1



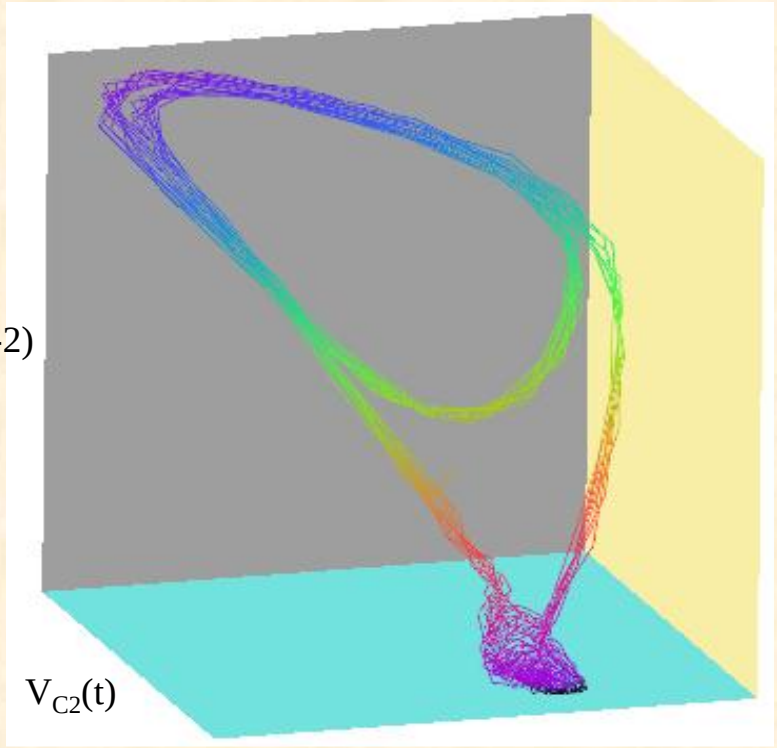
1^{∞} :R_1=1343Ω,
R_8=1831Ω,
R_C=100kΩ

Subcritical Hopf

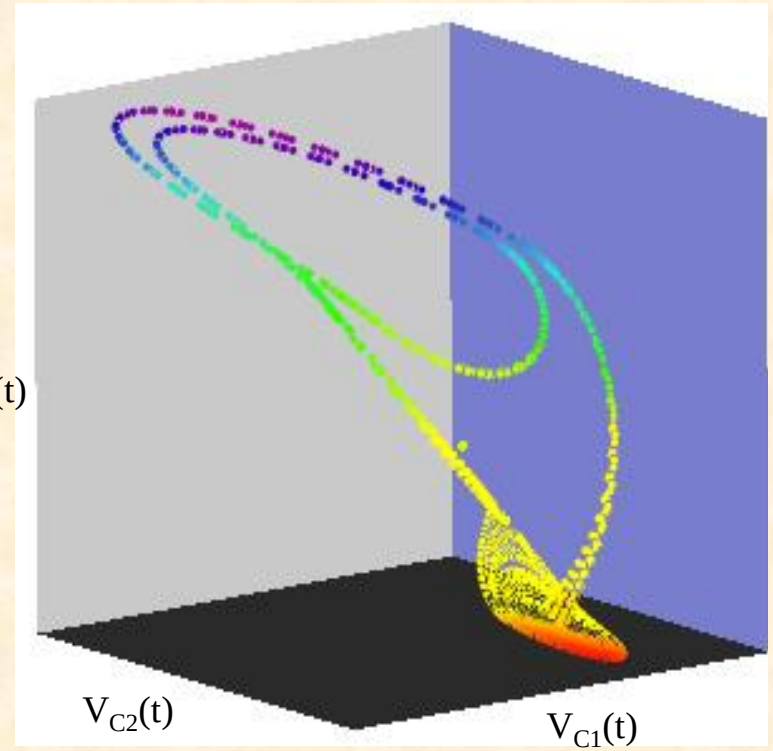
Shilnikov Chaos

2^{∞}: 3D Trajectory

Experiment

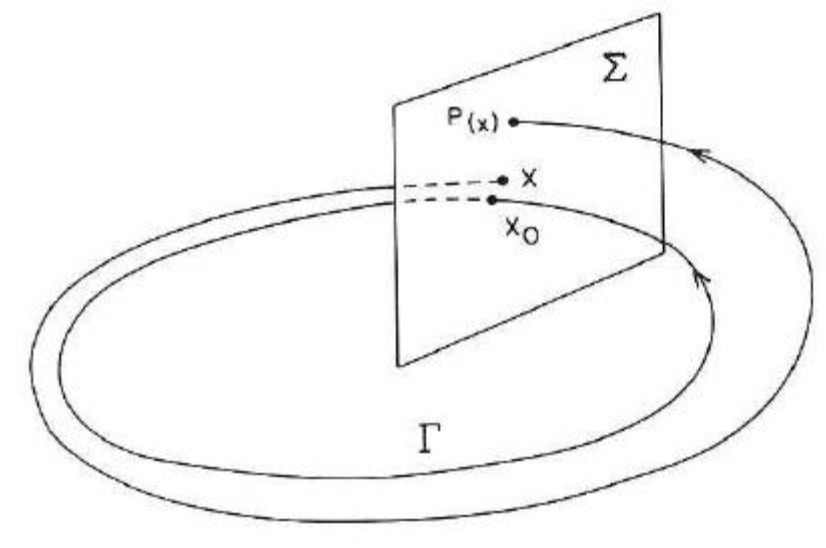


PSPICE

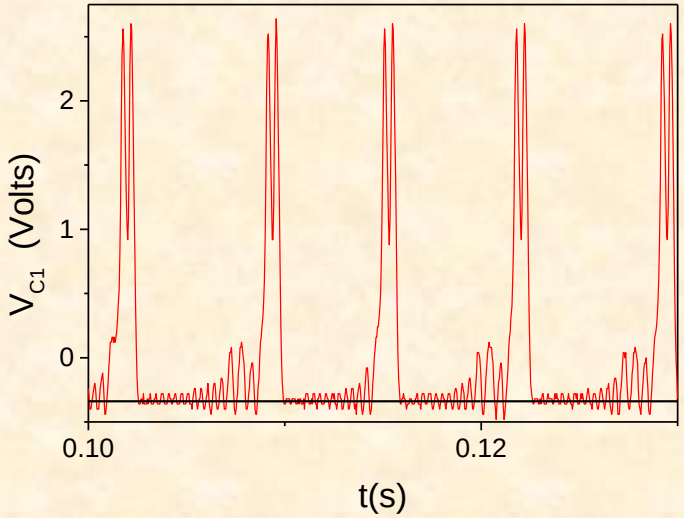


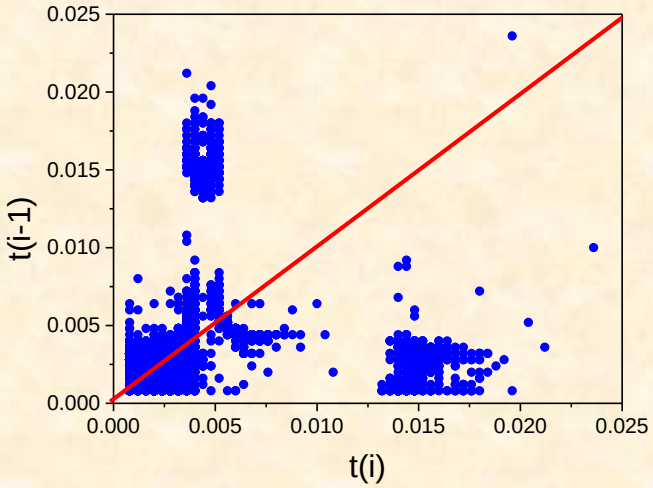
R_1=1431.95Ω, R_p=44.73kΩ, R_{23}=1580Ω, R_{25}=501.2Ω

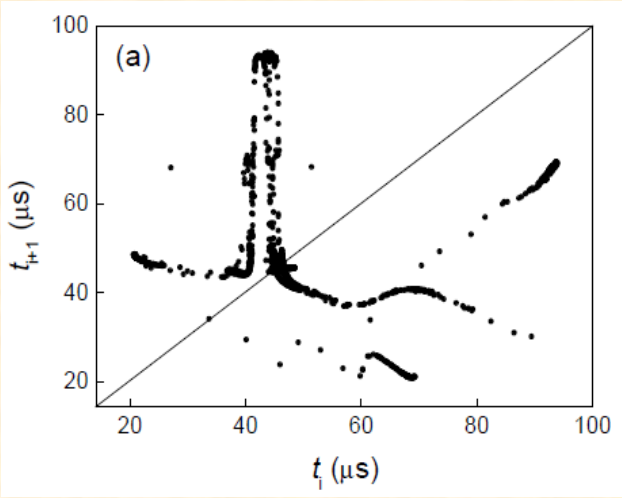
Poincaré Surface of Section



Shilnikov Chaos: Complexity



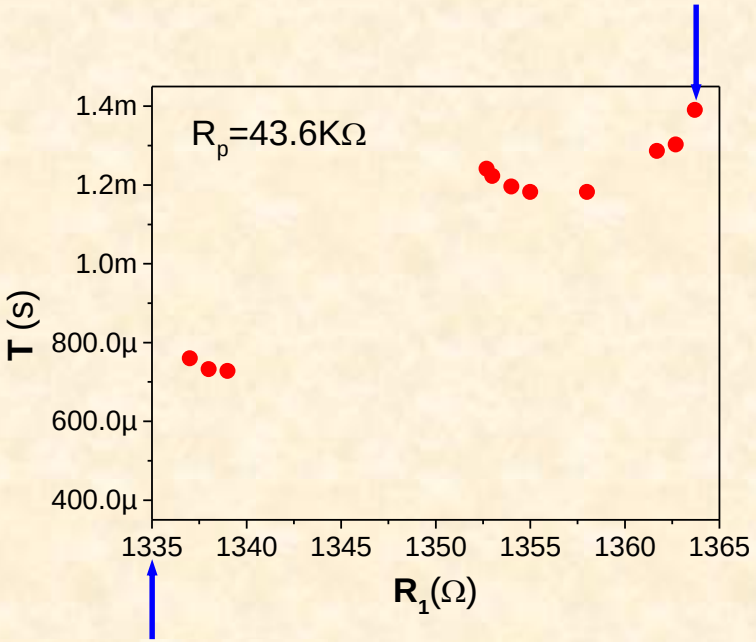




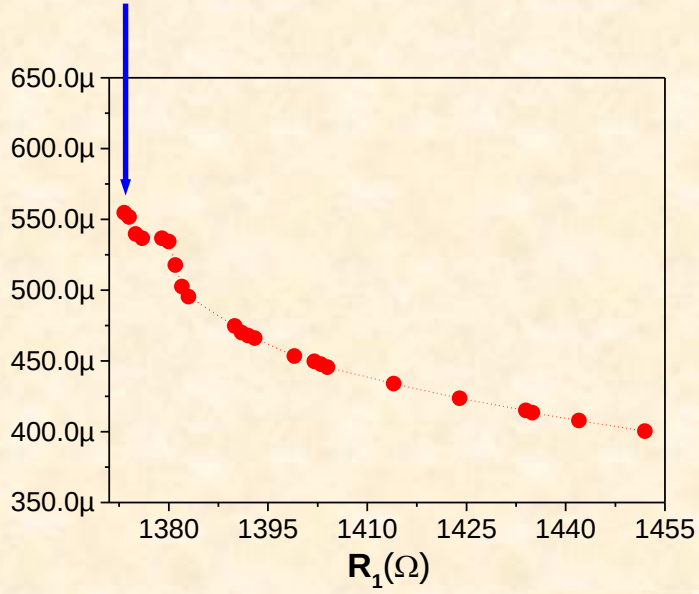
Dana, Chakraborty, Ananthakrishna, Pramana, **64(3),** 443 (2005)

Pisarchik, Meucci, Arecchi, *Eur. Phys. J.* D **13, 385 (**2001)

Period-Parameter Bifurcation



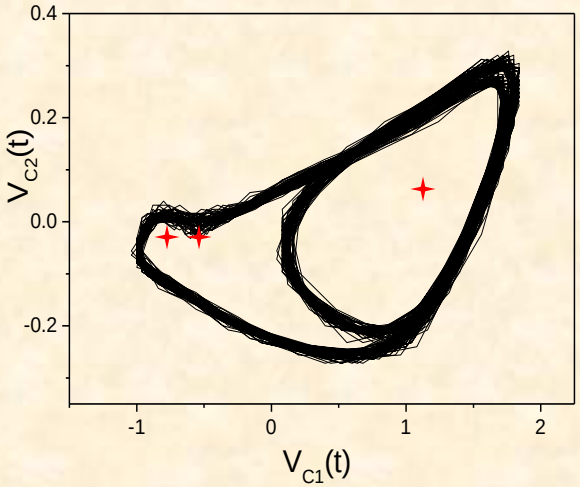
Subcritical Hopf

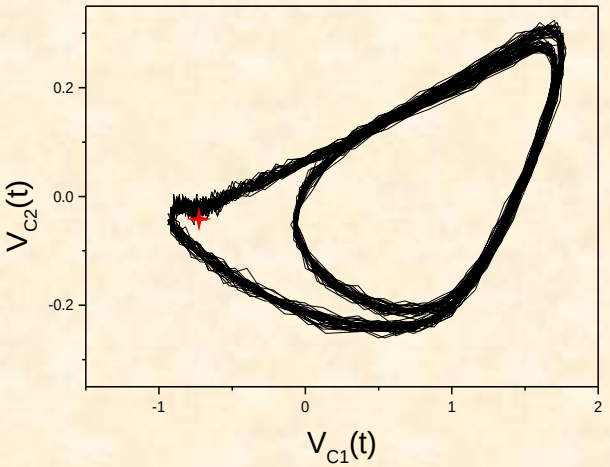


R_1=1524Ω

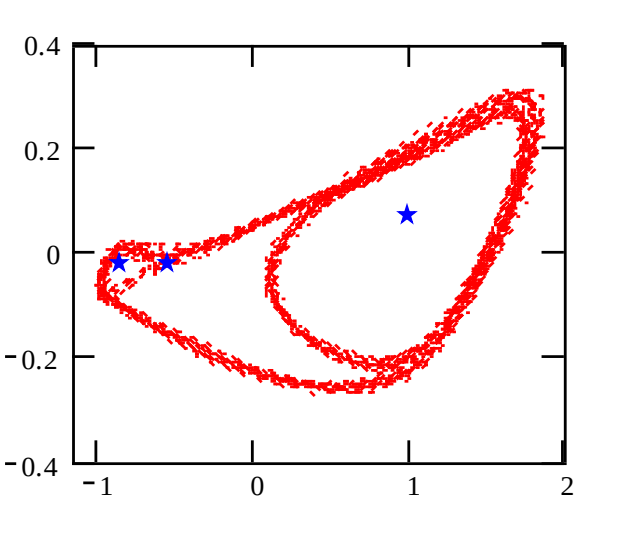
Supercritical Hopf

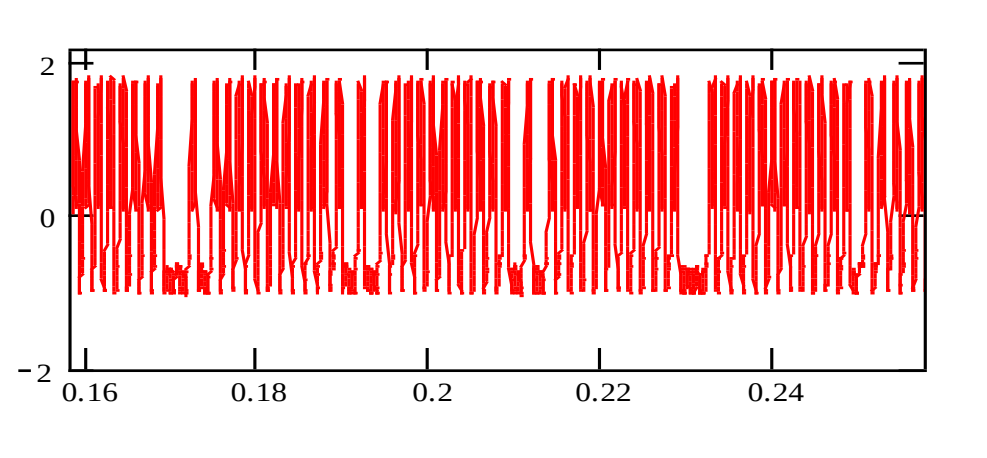
R_p= 43.6kΩ

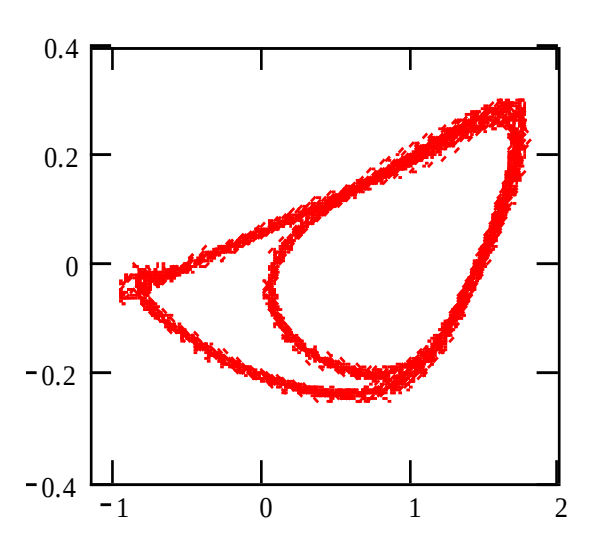


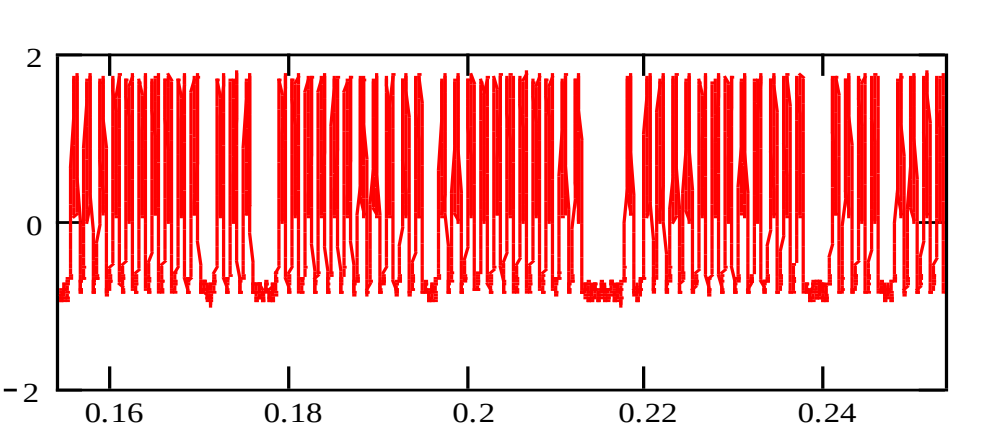


Chaotic Bursting

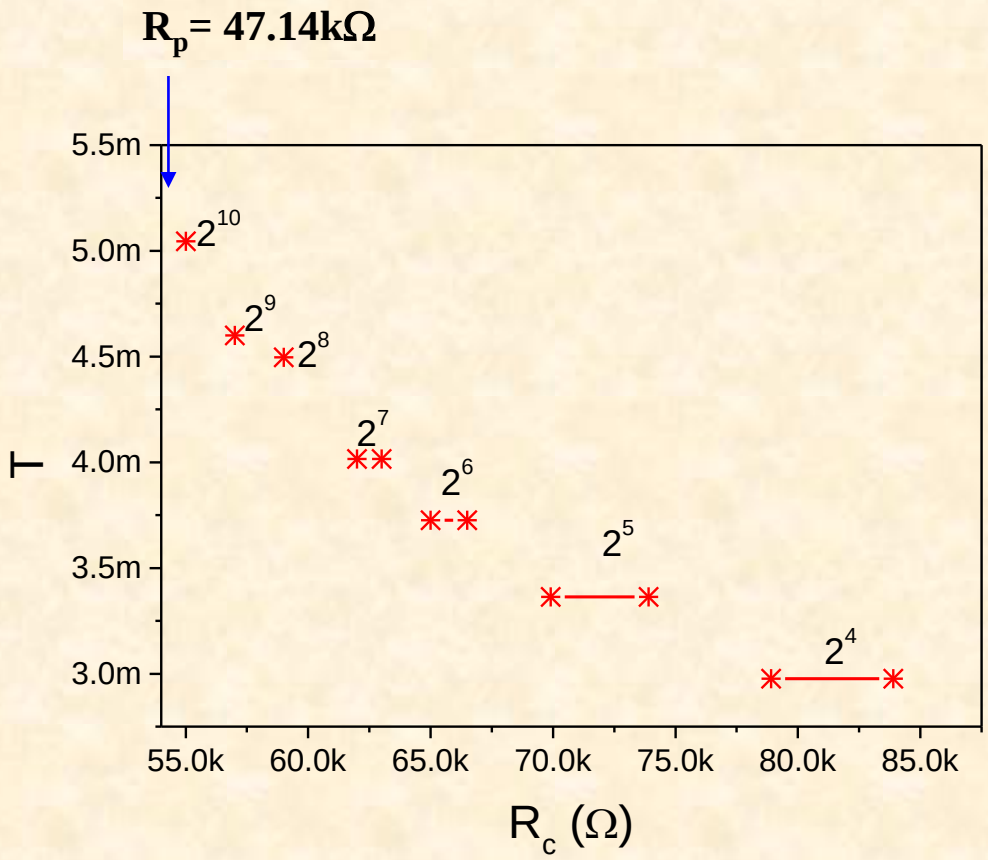


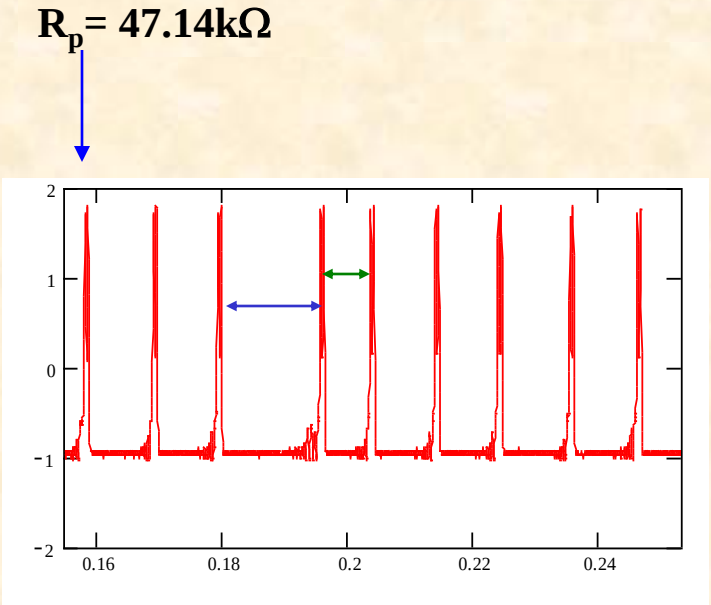






Period-Parameter Bifurcation



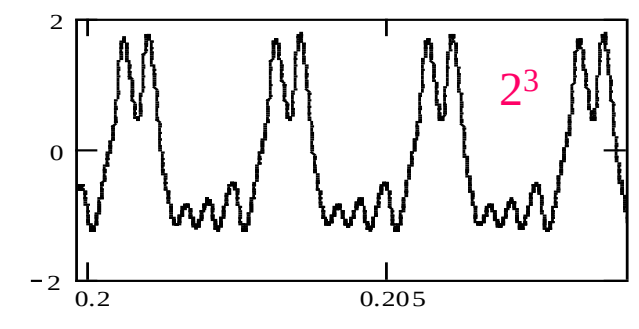


HC:2^{∞}

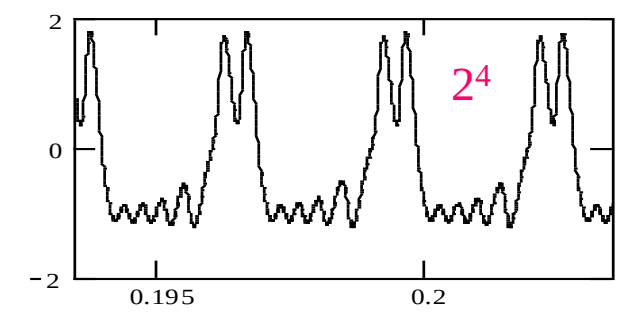
Δt_{max}=16ms

Δt_{min}=8ms

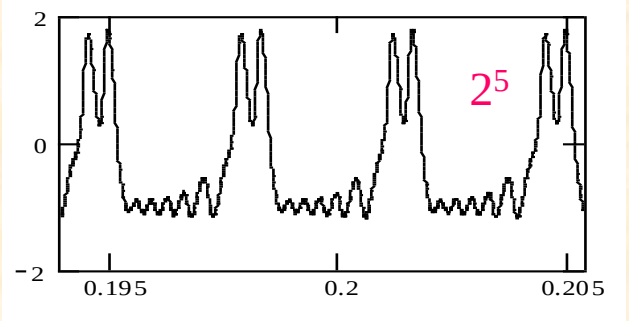
Mixed Mode Oscillations: 2^s



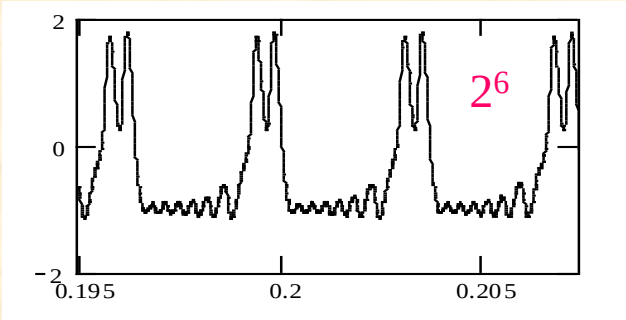
R_p=98.9k



R_p=83.9k



R_p=73.9k

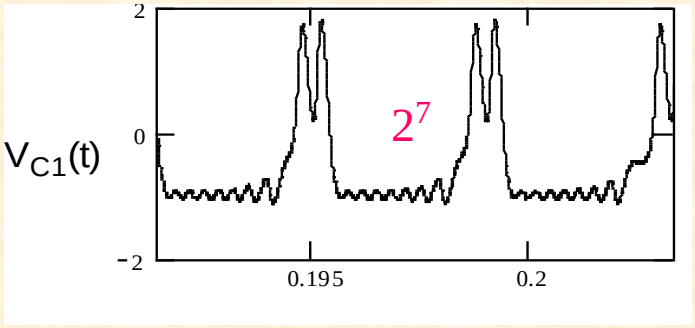


R_p= 66.5k

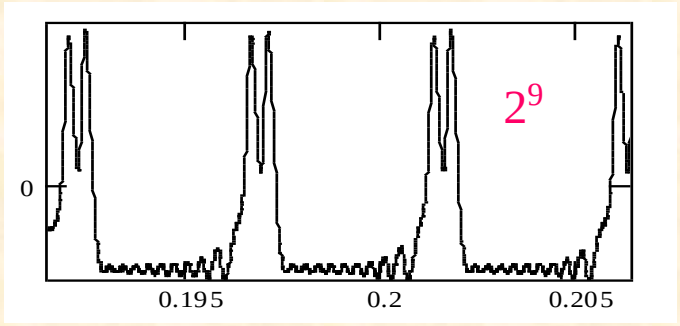
R_1=1357Ω, R_{23}=1.858kΩ, R_{25}=333Ω

P.Gaspard, X.-J.Wang, *J.Stat. Phys*., 48:151-199 (1987)
Marc.T.M.Koper, *Physica D*, 80, 72-94 (1995)
A.Goryachev, P.Strizhak, R.Kapral, *J.Chem..Phys*., 107:2881 (1997)
S.Rajesh, G.Ananthakrishna, *Physica D*, 140, 193-212 (2000)

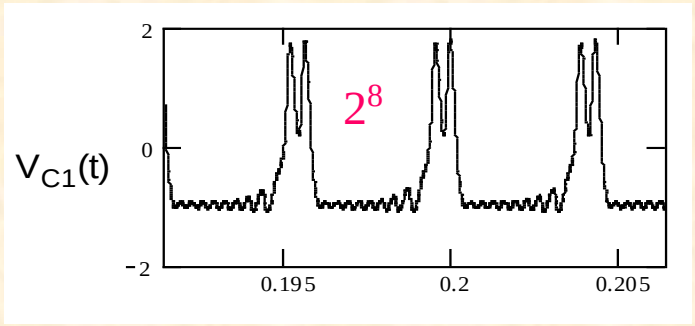
Mixed Mode Oscillations: 2^s



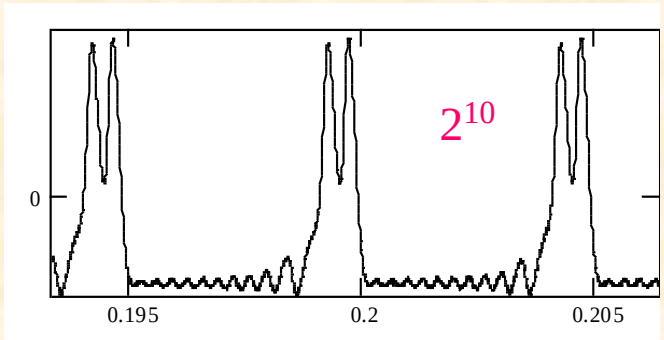
R_p=62kΩ



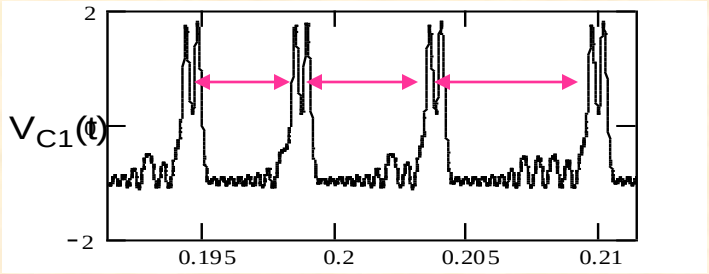
R_p=57kΩ



R_p=59kΩ



R_p= 55kΩ

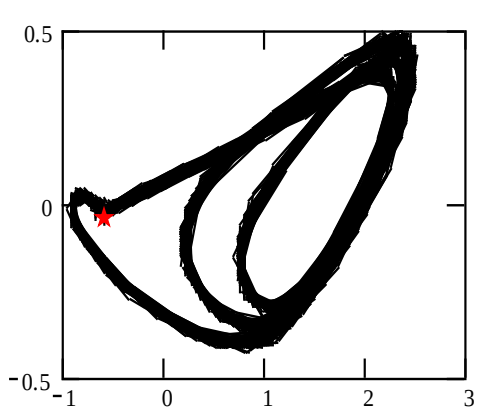


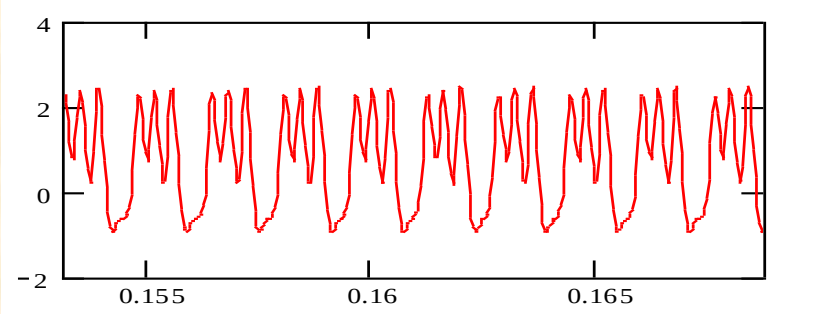
Unstable Orbit

R_1=1357Ω, R_{23}=1.858kΩ, R_{25}=333Ω

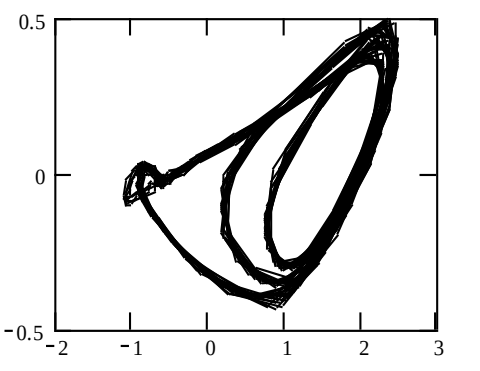
Homoclinic Chaos: 3^{∞}

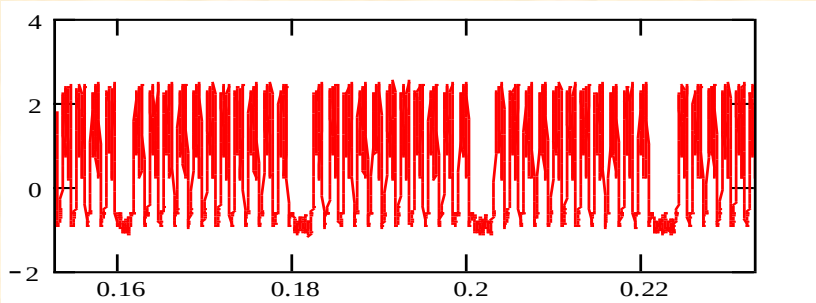
Homoclinic



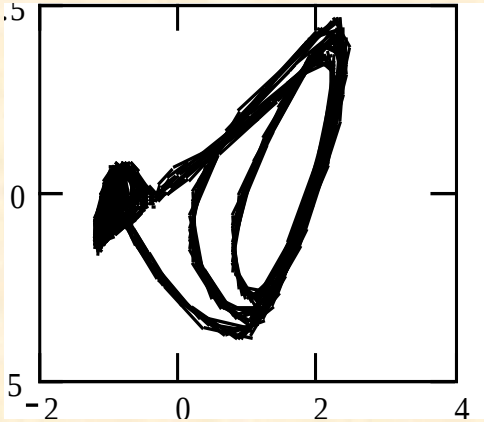


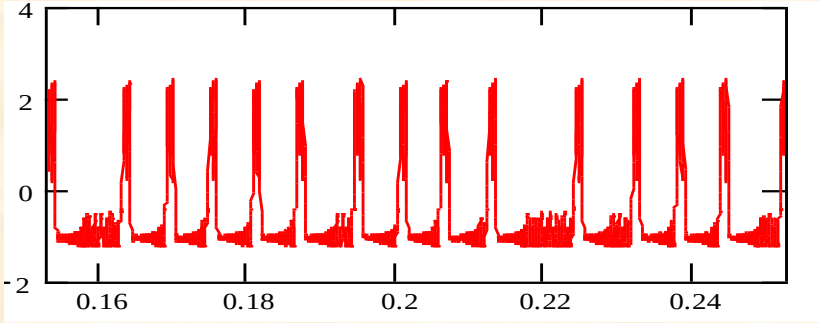
Bursting



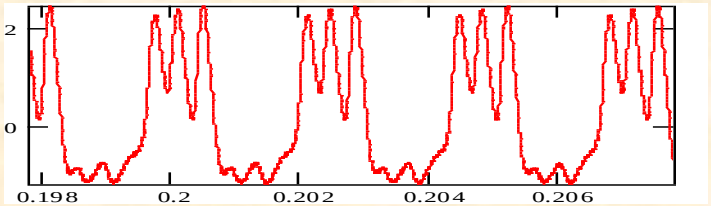


Homoclinic

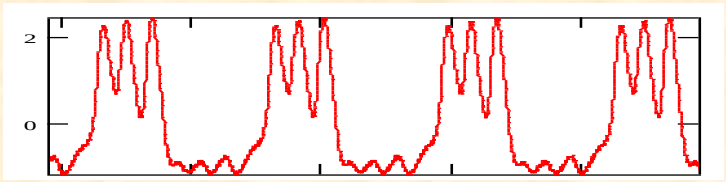




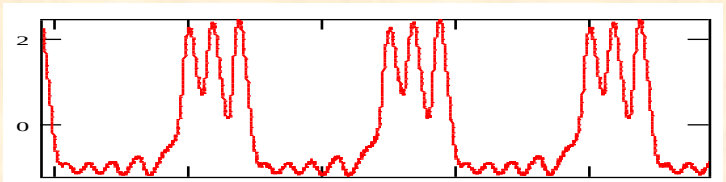
Mixed Mode Oscillations: 3^S



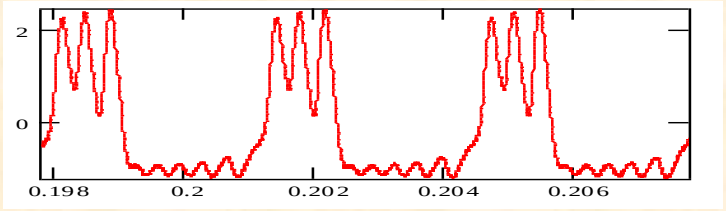
3^1



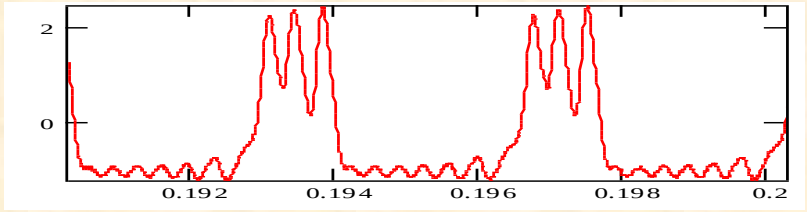
3^2



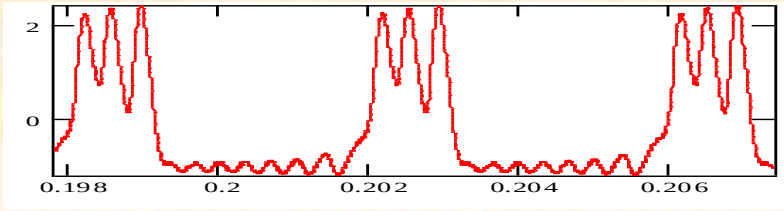
3^3



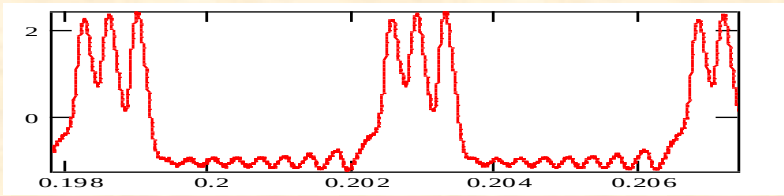
3^4



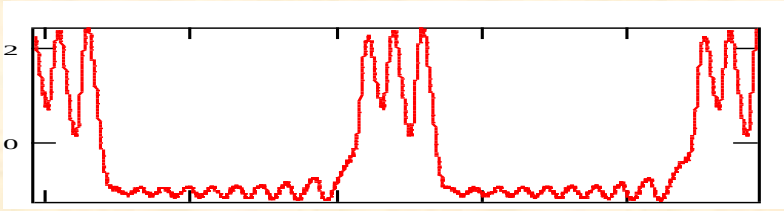
3^5



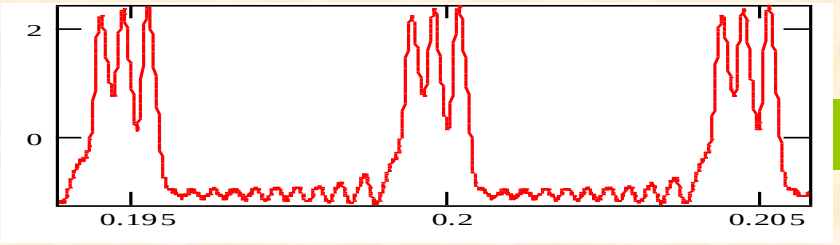
3^6



3^7

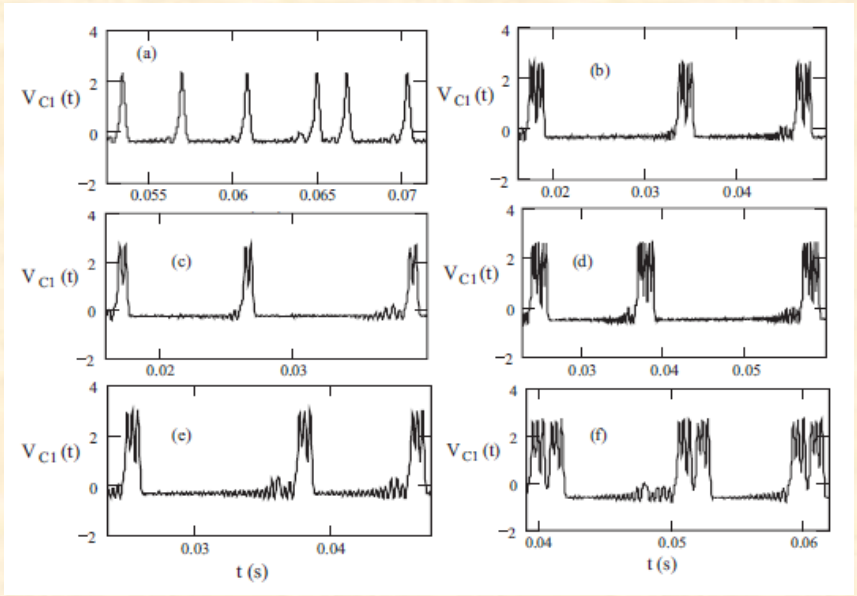


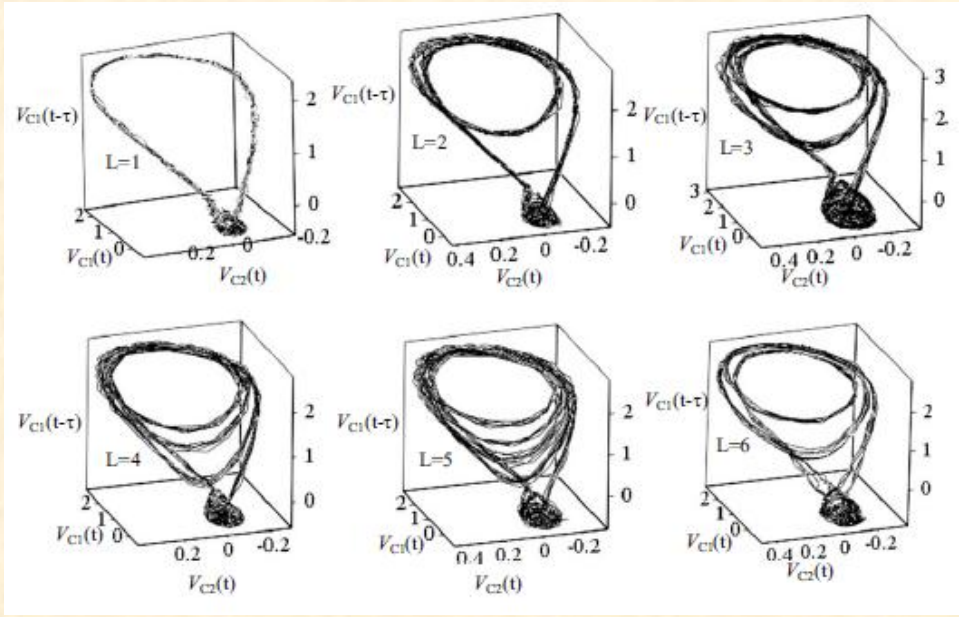
3^8



3^9

Homoclinic chaos: A Variety

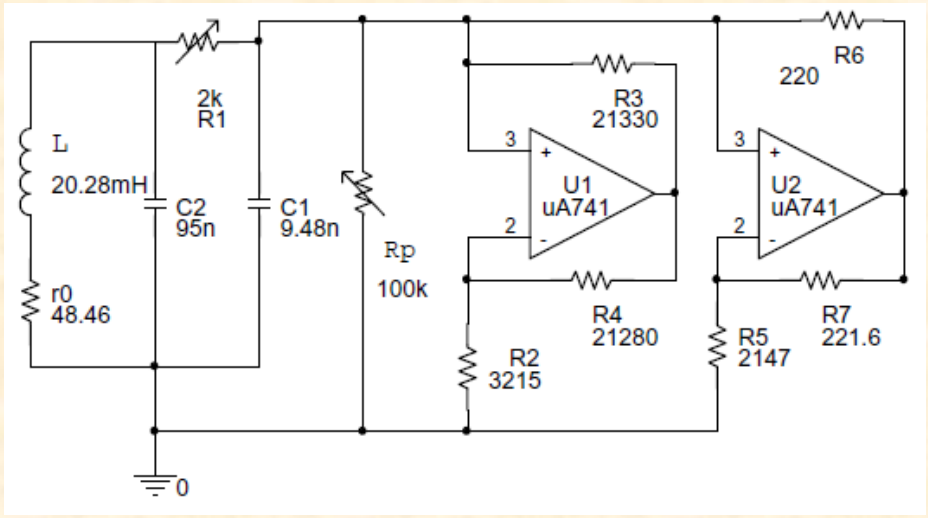


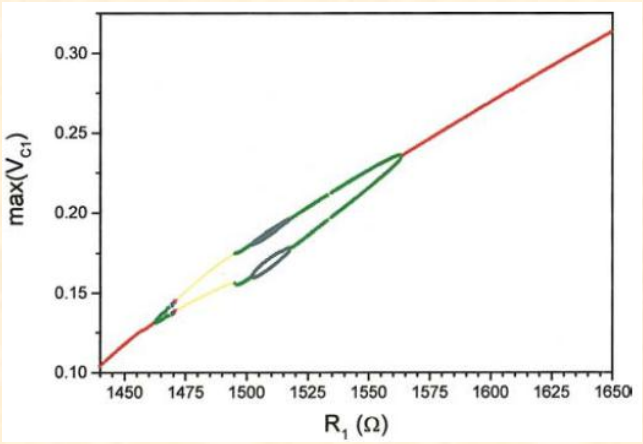


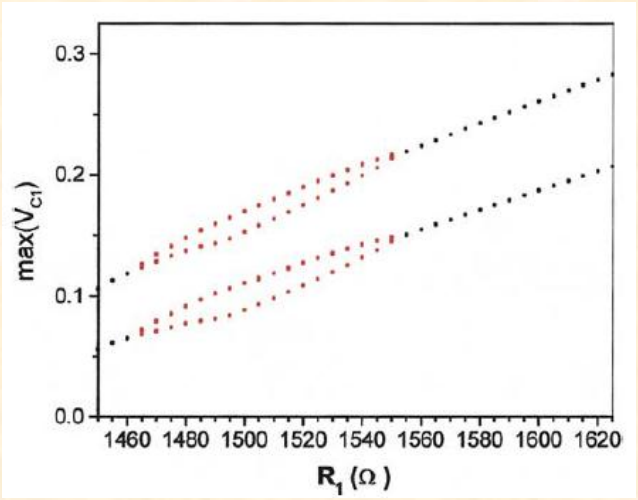
Homoclinic Chaos:
Video show

Gluing Bifurcation

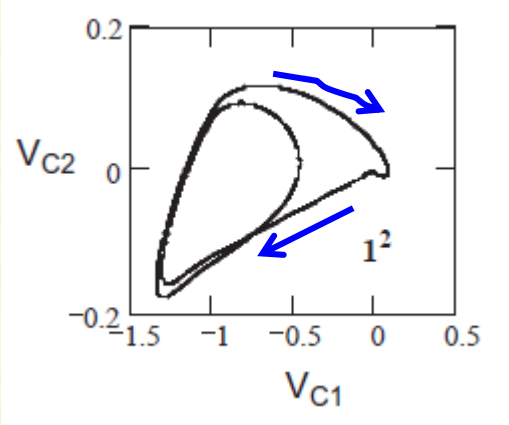
Modified Chua Circuit

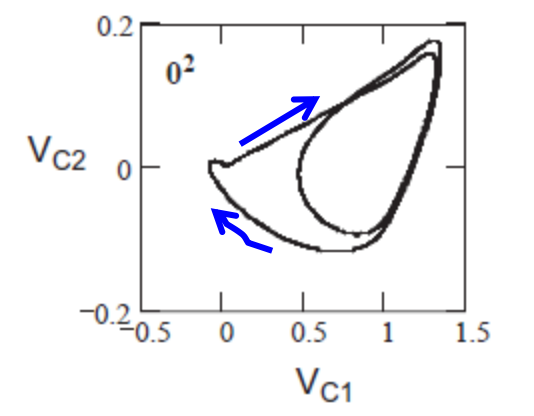




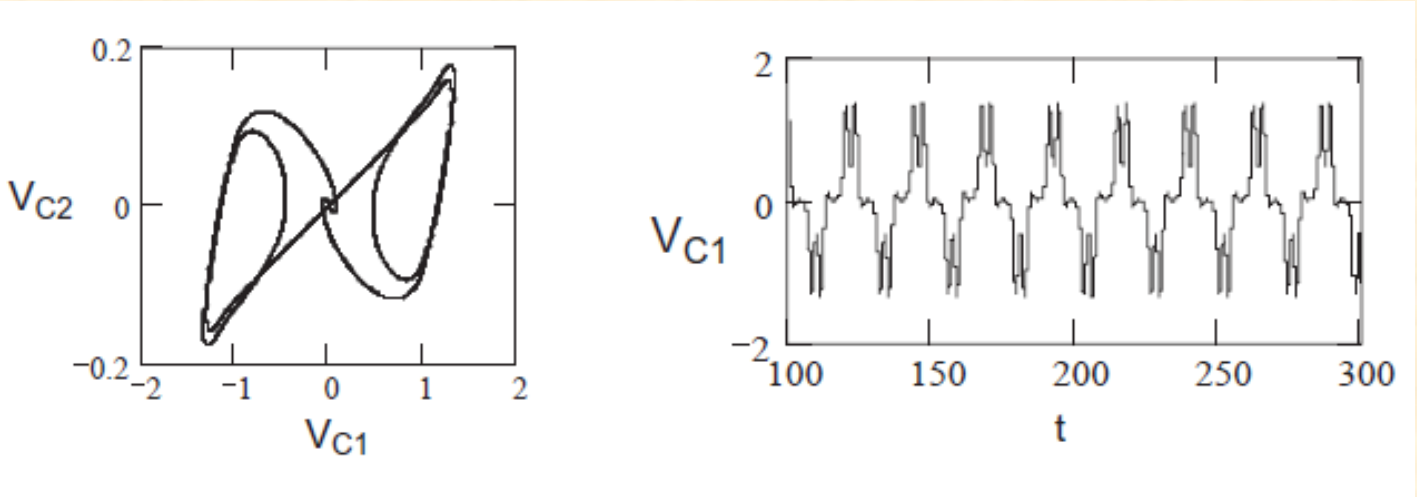


Coexisting Limit cycles

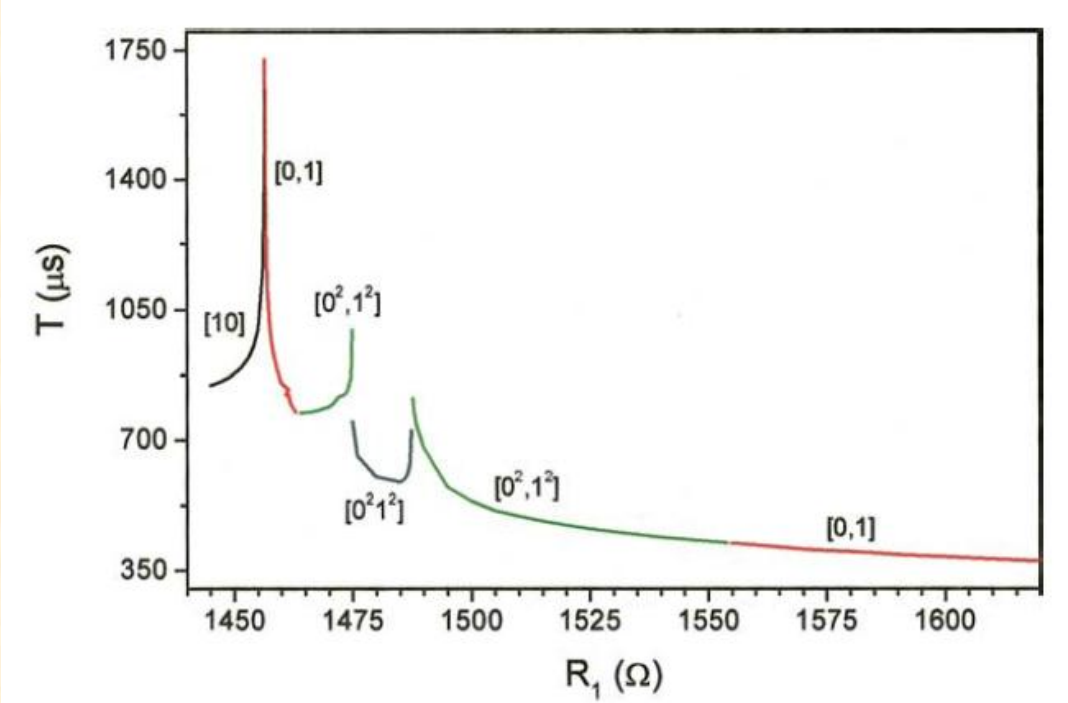




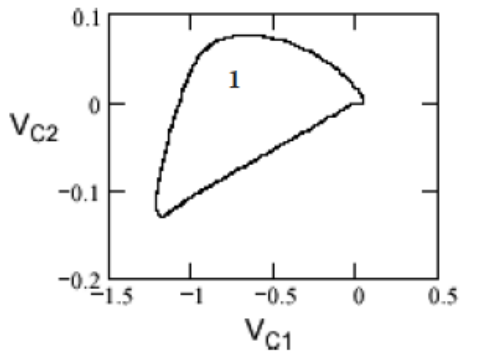
Gluing Bifurcation

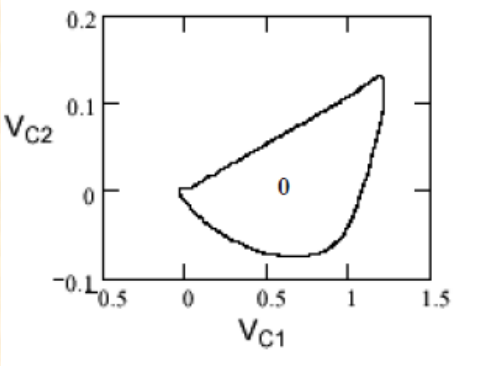


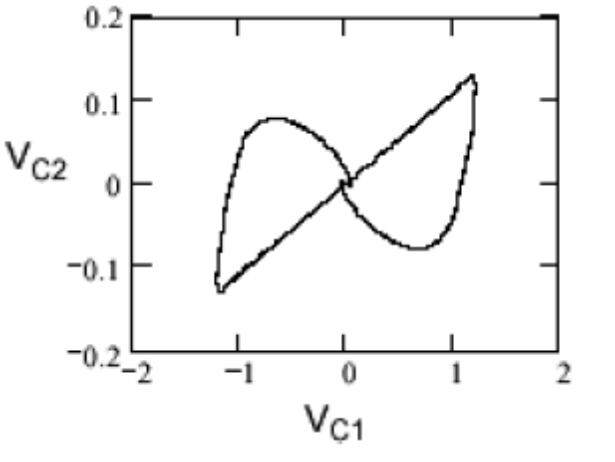
Period-parameter bifurcation



Gluing Bifurcation [1, 0]







Gluing Bifurcation:
MathCad Demonstration

Movie: Gluing.avi

Gluing Bifurcation

Directors:
Rupak Mukherjee, Prabhakar Srivastava

Photography:
Mitesh Patel, Unnati Patel

Music: Prabhakar

Producer:
Nonlinear Electronics Laboratory,
IPR, Gandhinagar

February 20, 2015