An Introduction to XPPAUT 7.0

HSND-2015, IPR

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XPPAUT=X-Windows Phase Plane plus AUTO

Developed by
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http://www.math.pitt.edu/~bard/xpp/xpp.html

How to install on Windows

If you don't already have an X-server, download and install :XMING. It is very easy to install and run!

Download **xppwin.zip** is a zipped file. Unzip it in the C:\ drive. Don't stick it in Programs or anywhere else unless you want to screw with the batch file. Unzipping it as recommended will produce a new directory called **C:\xppall**. Open this folder and make a shortcut to the Desktop of the file **xpp.bat**

It is now installed

How to run on Windows

Start your X-server

To use an existing file:

- Open the **xppall** folder on your C-drive
- Open the **ode** folder. There will be many examples
- With your mouse pick up, say, lecar.ode and drop it into the **xpp.bat** file on your Desktop. XPP should fire up with this ODE

To write your own:

- Open Notepad or something
- Type in an ODE.
- Save it as test.ode in the **xppall/ode folder** or anywhere else
- Drag and drop it into **xpp.bat**

What can be done?

Real time integration of
ODE
DDE
PDE
MAP
Phase plane plot

Bifurcation (Using AUTO interface)

Animation!!

A simple example

Write an ODE file for the Rossler attractor,

$$x' = -y - z,$$

$$y' = x + ay,$$

$$z' = bx - cz + xz,$$

with parameters $a = .36$, $b = 0.4$, and $c = 4.5$ and initial data $x = 0$, $y = -4.3$, and $z = 0$. Set the total integration time to 200. Make a three-dimensional plot by clicking on the little boxes next to the three variables in the **Initial Data Window**, and then click on the `xvsy` button in the **Initial Data Window**.

```
# rossler attractor
x1'=-y1-z1
y1'=x1+a*y1
z1'=b-c*z1+x1*z1
par a=0.36,b=.4,c=4.5
x1(0)=0
y1(0)=4.3
z1(0)=0
@ total=200,dt=0.01,runnow=1,maxstor=1000000
done
```

Lorenz attractor

```
x1'=s*(-x1+y1)
y1'=r*x1-y1-x1*z1
z1'=-b*z1+x1*y1
par d=0,q=.4,r=28,s=10,b=2.66
init x1=0.5,y1=0.1,z1=0.4
@ total=600,dt=.01,runnow=1,maxstor=100000
@ BUT=Quit:fq
done
```

Van der Pol Oscillator

```
x1'=y1
y1'=a*(1-x1^2)*y1-x1
x1(0)=0.2
y1(0)=0.3
par a=1.45
@ total=500,runnow=1,BUT=Quit:fq,
@ maxstor=100000,dt=.01
done
```

Initial condition→range

↓

To see a movie

Also use parameter/variable bar

```
# the classic logistic map
x(t+1)=a*x*(1-x)
par a=2.8
init x=.64285
@ total=200,meth=disc
done
```

Bifurcation diagram: Continuation

Pitchfork bifurcation

Supercritical
Subcritical

```
#super critical pitchfork bifurcation
x'=r*x-x^3
#subcritical pitchfork bifurcation
#x'=r*x+x^3
init x=.1
par r=-1
@ total=100
@ dt=.01,bound=1000000,maxstor=1000000
@ runnow=1
@ BUT=QUIT:fq
done
```

Saddle-Node bifurcation

```
#start with r=1
#x'=r-x^2
#start with r=-1
x'=r+x^2
init x=.1
par r=-1
@ total=400
@ dt=.01,bound=1000000,maxstor=1000000
@ runnow=1
@ BUT=QUIT:fq
```

Transcritical bifurcation

```
x'=r*x-x^2
init x=.1
par r=-1
@ total=100
@ dt=.01,bound=1000000,maxstor=1000000
@ runnow=1
@ BUT=QUIT:fq
```

Hopf bifurcation

```
x'=(a-x^2-y^2)*x-w*y
y'=w*x+(a-x^2-y^2)*y
x(0)=0
y(0)=0
par a=-2,w=2
@ total=200,runnow=1,maxstor=100000,dt=.01
@ BUT=Quit:fq
done
```

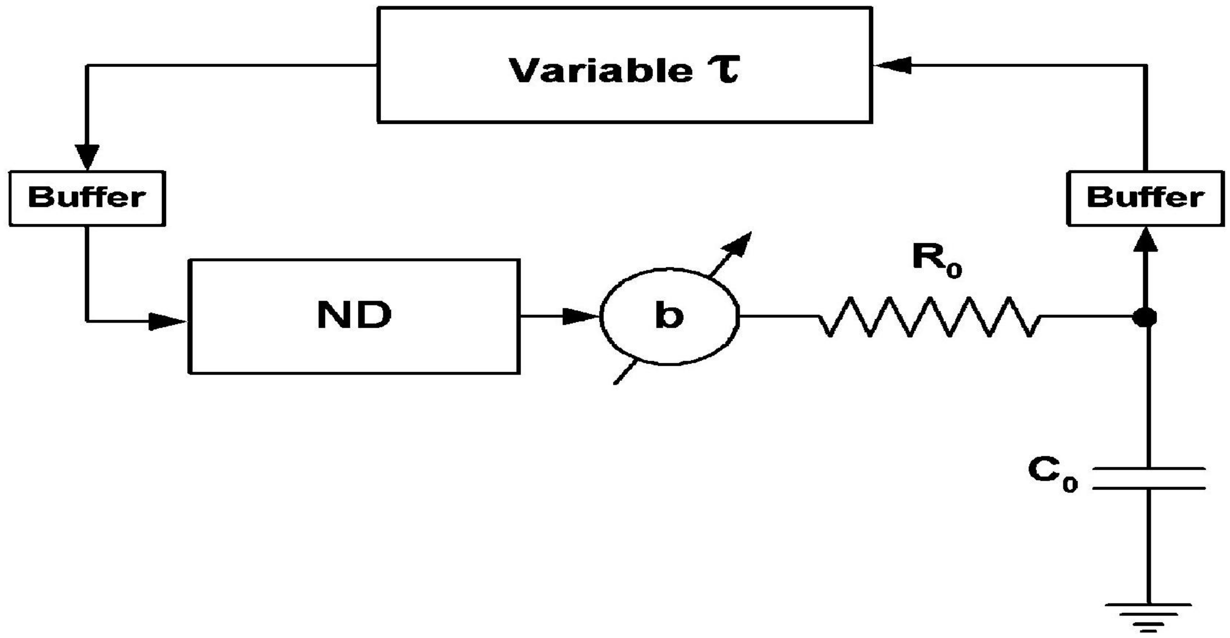
Delay Differential Equations

The delay differential equations in general may be given by:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t - \tau_i), \mu)$$

where, τ_i are all positive constants, called the time delays $i = 1, 2, \ldots, n$,

Time-delay hyperchaotic Electronic Circuits



T. Banerjee and D. Biswas, **Int. J. Bifurcation and Chaos**, Vol.21, 2013

T. Banerjee et al, **Nonlinear Dynamics**,Vol. 70 (1), pp. 721-734, 2012

Time delay Chaotic/hyperchaotic oscillator

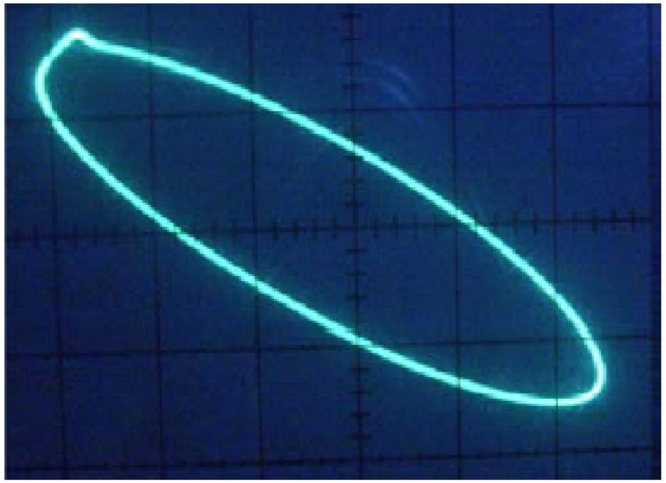
First-order retarded delay differential equation

$$\dot{x}(t) = -ax(t) + bf(x_\tau),$$

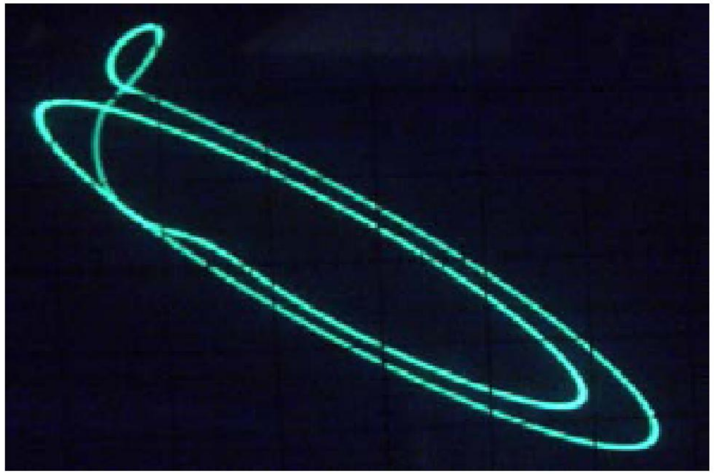
With the following nonlinearity

$$f(x_\tau) = -nx_\tau + m\tanh(lx_\tau),$$

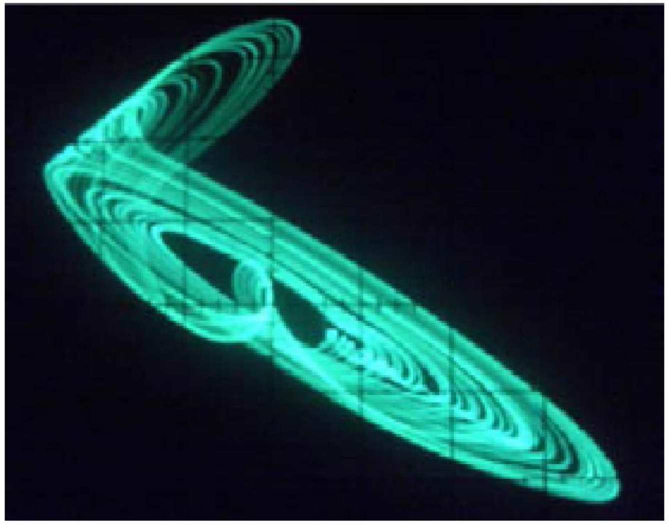
T. Banerjee and D. Biswas, **Int. J. Bifurcation and Chaos**, Vol.21, 2013



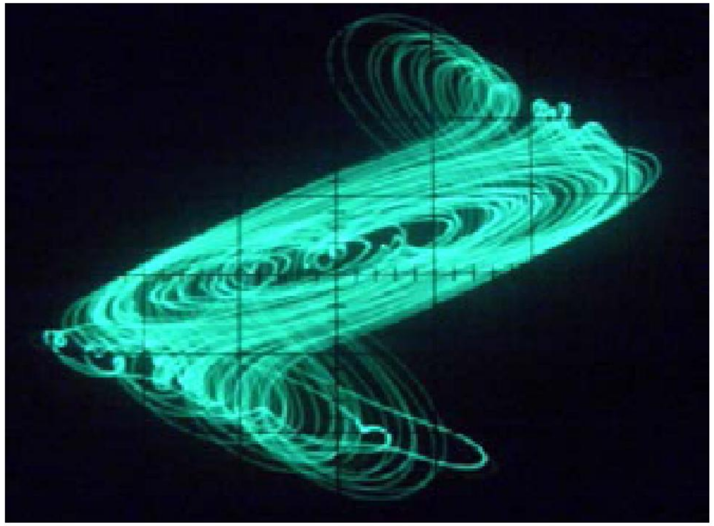
(a)



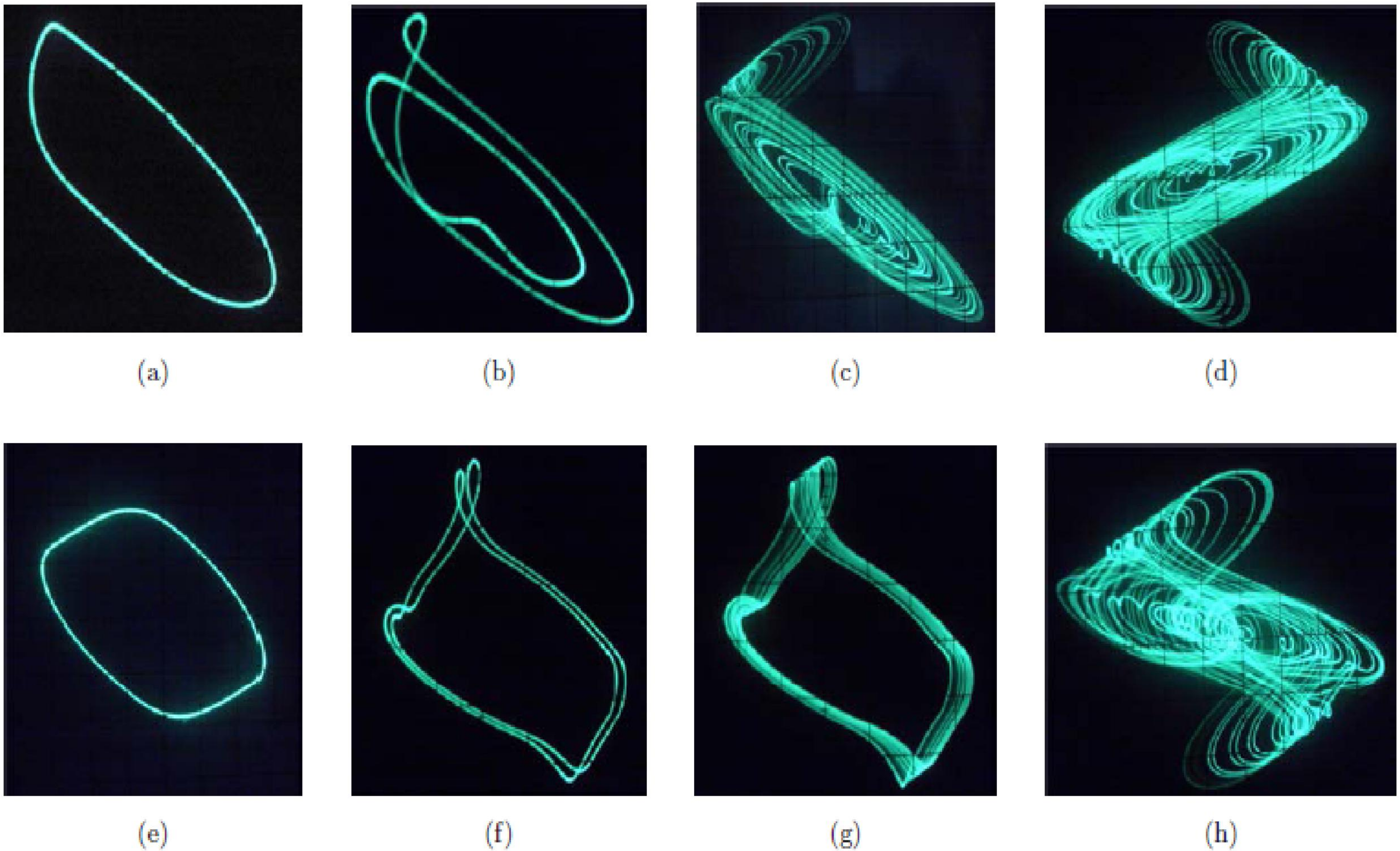
(b)



(c)



(d)


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)

DDE

```
f(x)=-n*x+m*tanh(l*x)
x'=-a*x+b*f(delay(x,tau))
aux y=delay(x,tau)
x(0)=.9
p a=1,tau=3.69,b=.96,n=2.2,m=1,l=10
@ delay=50
@ total=1100
@ dt=.01
@ T0=0
@ maxstor=1000000
@ bigfont=lucidasanstypewriter-bold-14
@ smallfont=lucidasanstypewriter-bold-14
@ but=quit:fq, transient=200
done
```

Lorenz System: Bifurcation Diagram

```
x1'=s*(-x1+y1)
y1'=r*x1-y1-x1*z1
z1'=-b*z1+x1*y1
Par r=0,s=10,b=2.66
init x1=0,y1=0,z1=0
@ total=200,dt=.01,runnow=1,maxstor=100000
@ BUT=Quit:fq
done
```

Coupled Oscillator: AD and OD

Diffusive coupling with no parameter mismatch: NO AD but OD is possible

```
x1'=(1-x1^2-y1^2)*x1-w1*y1+d*(x2-x1)
y1'=w1*x1+(1-x1^2-y1^2)*y1
x2'=(1-x2^2-y2^2)*x2-delta*w1*y2+d*(x1-x2)
y2'=delta*w1*x2+(1-x2^2-y2^2)*y2
x1(0)=0
y1(0)=0
x2(0)=0
y2(0)=0
par d=0,delta=1,w1=2,w2=2
@ total=300,runnow=1,BUT=Quit:fq
@ maxstor=100000,dt=.01
done
```

Coupled Oscillator: AD to OD

Mean-field coupling

```
#parameter mismatch is controlled by delta
(=w1/w2)
f(u,v)=(1-u^2-v^2)
x1'=f(x1,y1)*x1-w1*y1+d*(q*0.5*(x1+x2)-x1)
y1'=w1*x1+f(x1,y1)*y1
x2'=f(x2,y2)*x2-delta*w1*y2+d*(q*0.5*(x1+x2)-x2)
y2'=delta*w1*x2+f(x2,y2)*y2
x1(0)=0
y1(0)=0
x2(0)=0
y2(0)=0
par d=0,delta=1,q=.4,w1=2
@
total=300,runnow=1,BUT=Quit:fq,maxstore=100000,d
t=.01
done
```

Animation!!

With XPP

DASL: Dynamical Animation Scripting Language

dimension xlo;ylo;xhi;yho
speed delay
transient
permanent
line x1;y1;x2;y2;color;thickness
rline x1;y1;color;thickness
rect x1;y1;x2;y2;color;thickness
frect x1;y1;x2;y2;color
circ x1;y1;rad;color;thickness
fcirc x1;y1;rad;color draws a filled circle with radius **rad** centered at **(x1,y1)** with optional color.

$WHITE, $RED, $REDORANGE, $ORANGE, $YELLOWORANGE, $YELLOW, $YELLOWGREEN, $GREEN, $BLUEGREEN, $BLUE,$PURPLE, $BLACK.

Animation file showing ???

```
SPEED 20
#fcirc .5+x/40;z/50;.02;$GREEN
commet .5+x/40;z/50;-7;100;$GREEN
end
```

Filename.ani

sin(x)	$\sin(x)$	cos(x)	$\cos(x)$	tan(x)	$\tan(x)$
atan2(x,y)	$\tan^{-1}(y/x)$	asin(x)	$\sin^{-1}(x)$	acos(x)	$\cos^{-1}(x)$
atan(x)	$\tan^{-1}(x)$	sinh(x)	$\sinh(x)$	cosh(x)	$\cosh(x)$
tanh(x)	$\tanh(x)$	x**y,x^y	x^y	exp(x)	e^x
abs(x)	$\|x\|$	ln(x)	$\ln(x)$	log(x)	$\ln(x)$
log10(x)	$\log_{10}(x)$	sqrt(x)	$\sqrt{x}$	max(x,y)	$\max(x,y)$
min(x,y)	$\min(x,y)$	sign(x)	$x/\|x\|$	heav(x)	if $x \geq 0$ then 1 else 0
flr(x)	$\mathrm{int}(x)$	erf(x)	$\mathrm{erf}(x)$	mod(x,y)	x modulo y
erfc(x)	$1-\mathrm{erf}(x)$	x & y	x AND y	bessely(n,x)	$Y_n(x)$
x | y	x OR y	not(x)	~x	besselj(n,x)	$J_n(x)$

Double well potential

$$m\ddot{x} = x - x^3$$

$$\dot{x} = y, \quad \dot{y} = (x - x^3)/m.$$

$$E = my^2/2 + x^4/4 - x^2/2$$

```
# double well potential
x'=y
y'=(x-x^3)/m
par m=1
aux e=m*y^2/2+x^4/4-x^2/2
@ xp=x,yp=y,xlo=-1.5,ylo=-1.5,xhi=1.5,yhi=1.5
done
```

1. Click on `Graphic stuff Freeze On freeze`;
2. Click on `Initial conds mIce` and choose 10 or so initial conditions to get some nice trajectories;
3. Click on `nUmerics Colorize Another quantity` and choose `E` as the quantity and select `Choose`; set the minimum to $-.25$ and the maximum to $.75$; `Escape` to exit to the main menu;
4. Click on `Dir.field/flow Colorize` and choose a grid of 100;
5. Redraw by clicking `Restore`.