Abstract

In this thesis, Statistical Mechanics of two dimensional magnetised plasmas is studied. Major issues addressed are the following: (a) Is it possible to formulate the statistical mechanics of neutral and nonneutral plasmas confined in curved (toroidal) external magnetic field without any rotational transform, in such a way that the model is free from most of the problems encountered in the point vortex theory ? (b) Do the bifurcated solutions often found in linear devices (i.e., devices with uniform external magnetic field) confining nonneutral plasmas, exist in toroidal configurations. (c) It is well known that two dimensional (2D) models based on point vortex theory interact with long range forces. In the past, artificial models have been introduced to study the effect of finite interaction range. Is it possible to identify a magnetised plasma system in 2D, whose interaction range is finite and model it such that the defects of point vortex theory could be compensated ? (d) point vortex theories of current filaments have not been able to adequately explain the features of 'profile consistency' ideas often observed experimentally in Tokamaks, using methods developed in answering question (a) to (c), can a better model be proposed and solved ? We address the issues described above in detail.

In particular, we obtain the maximum entropy states of toroidal neutral and nonneutral plasmas obeying $\vec{E} \times \vec{B}$ drift dynamics confined by an *inhomogenous* external magnetic field $\vec{B}$ is formulated in terms of statistical mechanics of interacting *charge rings.* This becomes possible because in an ideal fluid, the relative velocity between a magnetic flux tube and the plasma fluid element is zero. Hence an axisymmetric flux tube with a non zero area of cross section can be identified as a *charge ring.* A maximum entropy problem is formulated in the general frame work of statistical mechanics of incompressible, non-overlapping vorticity regions and the sorrounding irrotational fluid. The *negative temperature transition* (which indicates the onset of formation of coherent structures) for toroidal neutral and nonneutral plasmas are obtained. We study neutral and nonneutral cases in two limits viz., a *point vortex limit* when the cross-section or thickness of the rings go to zero and a *hard-core limit*

where the cross-sections are finite and *nonzero.* In the case of neutral plasmas at point vortex limit, for large total electrostatic energy, we obtain nontrivial multiple solutions. In the case of nonneutral plasma in the same limit, we obtain the usual Liouville-like equation. We show that these equilibria are important to understand and interesting to study from the point of view of confining currentless toroidal nonneutral plasma at all energies and neutral plasmas when the total electrostatic energy is large. In the hard core regime, for neutral plasmas, we obtain what we have dubbed as "Tanh-Poisson equation" for chare rings. In the same regime, for nonneutral plasmas, we obtain a "Fermi-Poisson equation" for charge rings. In the nonneutral case, we have numerically solved the equations in cylindrical co-ordinates with conducting boundaries. This has been performed for various values of the 'hard core index' namely the *occupied fractional area* parameter α. The results bring into focus interesting new features of nonneutral plasma confinement in an external toroidal magnetic field. In order to answer the question as to whether there exists a hydrodynamic system that is *homologous* to charge rings, we formulate the statistical mechanics of interacting *vortex rings.* We find that unlike the charge and line vortices cases, the systems of charge and vortex rings do not have a homology. In otherwords, their mathematical forms, let alone the underlying physics, are qualitatively different. For example, the vortex rings behave more like current filaments and in general their equilibria are pushed towards larger major radial values whereas the charge ring equilibria have their equilibria pushed towards smaller major radial values. We also claim that our work is a natural generalization of the previous works within the assumed dynamics both in plasma as well as hydrodynamic contexts.

We apply the method developed using the concept of charge rings to a novel nonneutral plasma confinement device, where a poloidal magnetic field acts as the curved field which provides the radial confinement. Here, the confinement geometry enjoys translational invariance under linear displacement along the axial direction. Thus, an additional isolating integral of motion namely the total canonical linear momentum is introduced into the problem. The statistical mechanics of hard core charge rings is used to describe the long time state of this system. In the limit of point vorticity, we first explore the effect of this additional isolating integral of that of canonical linear momentum P. A notable result is that not all values of P can be accomodated in the system. This implies there are upper and lower bounds on P for a given total circulation. Further, it is well known that the onset of negative inverse temperature states, normally appears because of the *finiteness* of the phase space volume. Thus it is can be expected that when the volume of the phase space becomes infinite, the energy required to transit from positive to negative inverse temperatures also becomes infinite. Since our phase space is the space of magnetic flux, in the limit of tight aspect ratio ($\epsilon \longrightarrow 2$, where $\epsilon-$ inverse aspect ratio) the phase space volume becomes infinite. Thus the negative inverse temperature domain should vanish completely. But we find that depending on the value of the linear momentum constraint, even in the limit $\epsilon \longrightarrow 2$ we find that the transition energy can remain *finite*.

We have obtained *symmetry-breaking* long-time states in the *inhomogeneous* magnetic field in the point vortex limit. These bifurcating nonlinear states are interpreted as arising due to *diocotron instability* or *slipping stream instability* in curved magnetic field. Again, we show that our work reduces to that of previous works in the limit of zero magnetic field inhomogeneity. We have performed *inverse aspect ratio* studies and *periodic length studies.* Within the framework of the basic $\vec{E} \times \vec{B}$ dynamics, the entropic studies appears to indicate that *above* certain energy threshold, *shear layers* are *unstable* and there is a *stability transfer* from *shear layers* to *convective cells.* We have solved numerically for the bifurcation point in the hard core limit as well. This study was motivated by the question as to what is the effect of hard core on the bifurcation point. In answering the question we have performed the bifurcation study for various values of the hard core parameter q* which is nothing but the maximum vorticity value. We find that the scaling $\beta^{pvl} = q^* \ \beta^{hcl}$ is confirmed. Further we have obtained an scaling expression for entropy in the point vortex limit in terms of the entropy in the hard core limit as $S^{pvl} = q^* \ S^{hcl} - Q - Q \ln{(q^*)}$ where Q is the total charge. We numerically verify these expressions at the bifurcation energy value which is independent of the whether the limit is point vortex or hard core.

Both ours and that of other workers' models suffer from the problem related to the nature of *long-range interactions*. Hence, computation of various equilibrium and transport properties have undesirable and unphysical features like dependence of the transport coefficients on the system size, absence of thermodynamic limit, etc. In the past, there have been various models wherein, the range of interaction is *artificially truncated*, thereby eliminating such undesirable features as described above. It becomes important to study a realizable 2D system with finite interaction and compute various physical quantities described above, so that a comparision with long range models becomes plausible. To this end, we use the equations of Reduced Electron Magnetohydro Dynamics [REMHD] and reformulate them in terms of statistical mechanics of interacting generalized vortices with hard core. The interaction range of this system is logarithmic at short distances and falls of exponential for large distances. We obtain the *equilibrium* and *equation of state & transport coefficients* of this system and compare it with the long-range models. The results show that all the above said undesirable features of the previous models are indeed rectified to a reasonable extent. We compute numerically solutions corresponding to zero, one and two extrema of the field quantities with the help the a novel numerical algorithm was originally developed for 2D hydrodynamic studies. Entropy calculation of these states show that the entropy of one extrema distribution is largest compared to that of zero extremum (1D) solutions or two maxima [or in general] any multiple extrema solutions. We have compared our results of maximum entropy states in the case of nonzero, generalized vorticity plasma, with the numerical simulation studies of Bulanov, wherein, their long-time states show profiles with two extrema. Therefore from our studies, we conclude that either their states are not completely saturated or their dynamics is trapped in one of the many metastable states possible in a nonlinear time evolution.

Finally, we formulate the statistical mechanics of current filaments of *finite* cross-section where the total number of filaments as well as the total current in the given volume are specified. Unlike the previous formulations, our formulation respects all the physical quantities conserved by the underlying dynamics. We apply this model to 'predict' *profile consistency* and obtain meaningful results for the current density profiles. Unlike previous results which yield nonzero current density values on the wall, our theory provides a natural control parameter, namely the strength of the current filaments, which restricts the current density at the wall to zero. This feature comes as a consequence of conserving total current and the initial fractional current occupying area which in turn would decide the strength of the current filament.

To conclude, we have studied, through methods of statistical mechanics, long-time states of magnetised, two-dimensional plasmas. Main emphasis is on improving the already existing models in the case of homogenous magnetic systems and to develop altogether new models for inhomogenous magnetic systems, both for the case of neutral and nonneutral plasmas. Wherever possible we compare them with their hydrodynamic counterparts.