

# Seminar

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## Institute for Plasma Research

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**Title:** Nonlinear Mixing and Synchronization in Dusty Plasma  
**Speaker:** Dr. Ajaz Ahmad Mir  
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**Date:** 6<sup>th</sup> September 2023 (Wednesday)  
**Time:** 03:30 PM  
**Venue:** Seminar Hall, IPR

### Abstract

Dusty plasma is a collection of heavily charged particles immersed in a typical electron-ion plasma. It supports dust acoustic waves (DAWs) in its fluid regime due to the balance between dust inertia and the background plasma pressure. This thesis provided a theoretical model for nonlinear mixing and synchronization of DAWs using a forced Korteweg-de Vries (fKdV) model and a forced Korteweg-de Vries-Burgers (fKdV-B) model, respectively. Results from this thesis are in excellent agreement with experimental observations<sup>1,2</sup>.

The nonlinear dynamical evolution of dusty plasmas as a charged fluid is governed by fluid-Poisson equations. Solving them analytically and numerically is hard, leading to limited visualization of physical phenomena. However, in a weak nonlinear dynamical regime, these coupled fluid-Poisson equations reduce to KdV and KdV-B equations with and without the viscous damping, respectively. These models are convenient to solve and provide good visualization of DAWs and other nonlinear longitudinal collective modes with sufficient accuracy.

Nonlinear mixing is a phenomenon in which the nonlinear interaction of two or more modes generates a cascade of coherent modes. The characteristic features of the mixed-mode spectrum primarily depend on the nature of nonlinearity and dispersion in the medium. Using the fKdV model<sup>3</sup>, we theoretically model the nonlinear mixing in dusty plasma. The fKdV-based model shows excellent agreement with the experiment done by Nosenko et al.<sup>1</sup> on the mixing of compressional acoustic waves in a dusty plasma. The origin of modes in the spectrum is due to the three-wave mixing confirmed using the bispectral analysis<sup>4</sup>.

Synchronization is a phenomenon in which a weak nonlinear interaction between the natural mode and an external driver adjusts the natural mode's rhythms. This thesis has established that the fKdV-B equation provides a proper theoretical framework to explain synchronization in dusty or other dispersive plasmas. We also conclude that the dissipative effects are important to achieve the limit cycle, i.e., synchronization. The synchronization domains are delineated in the form of Arnold's tongue in the two-dimensional parametric space of the forcing frequency and forcing amplitude. Results from the fKdV-B equation-based model<sup>5</sup> showed good agreement with the synchronization experiments done by Ruhunusiri et al.<sup>2</sup>.

This thesis provides a simple, efficient, and accurate theoretical description of nonlinear mixing and synchronization of acoustic dust waves in agreement with previous experiments. Our model is extendable to similar nonlinear waves plasma at other lengths and time scales and fluid-like mediums with convective nonlinearity and dispersion.

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1. Nosenko, V. and Avinash, K. and Goree, J. and Liu, B.: Nonlinear Interaction of Compressional Waves in a 2D Dusty Plasma Crystal. *Phys. Rev. Lett* **92**, 085001, (2004)

2. Ruhunusiri, S. D. W. and Goree, J.: Synchronization mechanism and Arnold tongues for dust density waves. *Phys. Rev. E* **85**, 046401 (2012)

3. Mir, A. A. and Tiwari, K. S. and Goree, J. and Sen, A. and Crabtree, C. and Ganguli, G.: A forced Korteweg-de Vries model for nonlinear mixing of oscillations in a dusty plasma. *Phys. Plasmas* **27**, 113701 (2020)

4. Mir, A. and Tiwari, S. and Sen, A.: Bispectral analysis of nonlinear mixing in a periodically driven Korteweg-de Vries system. *Phys. Plasmas* **29**, 032303 (2022)

5. Mir, A. and Tiwari, S. and Sen, A. and Crabtree, C. and Ganguli, G. and Goree, J.: Synchronization of Dust Acoustic Waves in a forced Korteweg-de Vries-Burgers model. *Phys. Rev. E* **107**, 035202 (2023)

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